Politechnika Poznańska
Wydział Inżynierii Lądowej i Transportu
Instytut Analizy Konstrukcji
Zakład Mechaniki Budowli

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Rok akademicki: **2023/24**
Grupa ćw. / Grupa proj.: **C1 / P1**

MECHANIKA BUDOWLI / SEM.4
ĆWICZENIE NR 2

DYNAMIKA - UJĘCIE KLASYCZNE

Termin oddania: 13.06.2024

Data	Uwagi sprawdzającego	Podpis

~~ opracowanie: mgr inż. Anita Kaczor ~~ https://anita.kaczor.pracownik.put.poznan.pl ~~

Dla zadanej **BELKI** oraz **RAMY:**

1. Obliczyć częstości drgań własnych i sprawdzić, czy układ jest w strefie rezonansowej. Narysować postacie drgań własnych i sprawdzić ich ortogonalność.
2. Obliczyć amplitudy drgań punktów zaczepienia mas, siły bezwładności oraz obwiednię momentów zginających wywołane zadaną siłą dynamiczną.
3. Sprawdzić maksymalne naprężenia normalne, uwzględniając dwa schematy obciążenia: a) obciążenie dynamiczne oraz b) obciążenie statyczne (ciężar mas skupionych).

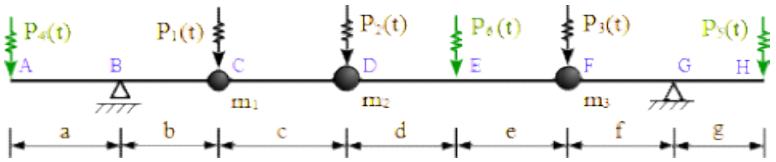
W obliczeniach przyjąć: E = 210 GPa, f_y = 235 MPa.

DANE DLA BELKI **Nr schematu: 2**

m_1	m_2	m_3	a	b	c	d	e	f	g	h	przekrój
[kg]	[kg]	[kg]	[m]	[m]	[m]	[m]	[m]	[m]	[m]	[m]	
400	**450**	**600**	**2,5**	**3,2**	**3,2**	**2,5**	**1,7**	**3**	**2,1**	**0**	**I 100 PE**

Na belkę działa siła **nr 2**, zmienna w czasie zgodnie z zależnością: $P_2(t) = P\sin(pt)$

- amplituda siły P [kN]: **3,5**
- częstotliwość siły p [Hz]: **23,7**

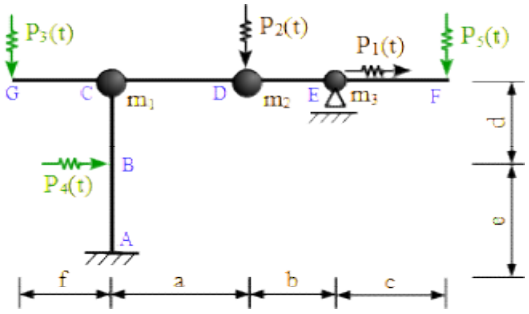


DANE DLA RAMY **Nr schematu: 4**

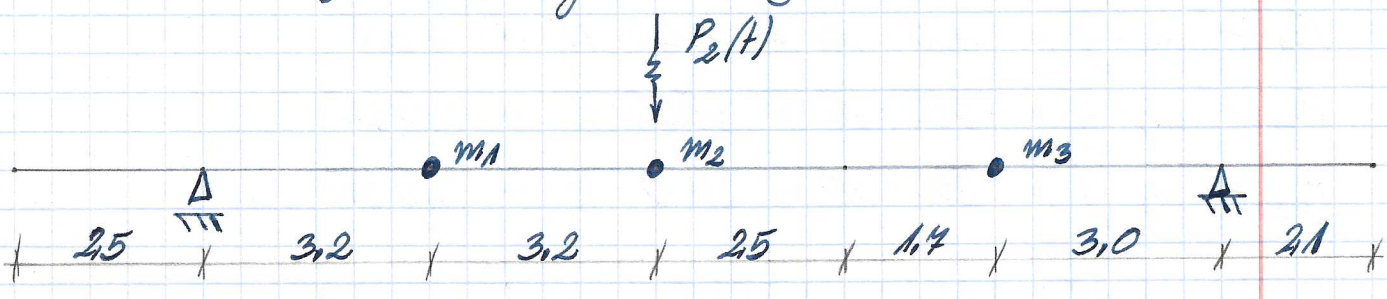
m_1	m_2	m_3	a	b	c	d	e	f	przekrój
[kg]	[kg]	[kg]	[m]	[m]	[m]	[m]	[m]	[m]	
400	**350**	**950**	**1,8**	**2,3**	**2,5**	**3**	**2,4**	**3**	**I 100 PE**

Na ramę działa siła **nr 2**, zmienna w czasie zgodnie z zależnością: $P_2(t) = P\cos(pt)$

- amplituda siły P [kN]: **3,4**
- częstotliwość siły p [Hz]: **24,5**



Dynamika – ujęcie klasyczne – Belka



SSD = 3

Drgania własne:

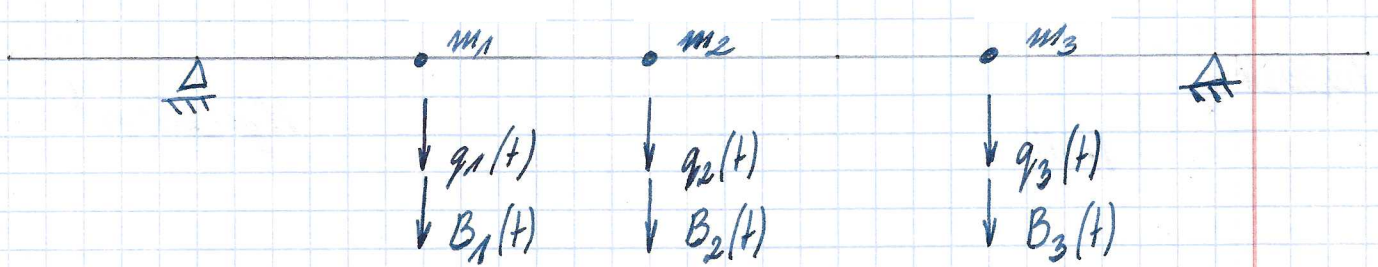
$q_i(t) = a_i \sin \omega t$

$P_i(t) = -m_i \ddot{q}_i(t)$

$\ddot{q}_i(t) = -a_i \omega^2 \sin \omega t$

$B_i(t) = -m_i \ddot{q}_i(t)$

$B_i(t) = m_i a_i \omega^2 \sin \omega t$



$$\begin{cases} q_1(t) = \delta_{11} B_1(t) + \delta_{12} B_2(t) + \delta_{13} B_3(t) \\ q_2(t) = \delta_{21} B_1(t) + \delta_{22} B_2(t) + \delta_{23} B_3(t) \\ q_3(t) = \delta_{31} B_1(t) + \delta_{32} B_2(t) + \delta_{33} B_3(t) \end{cases}$$

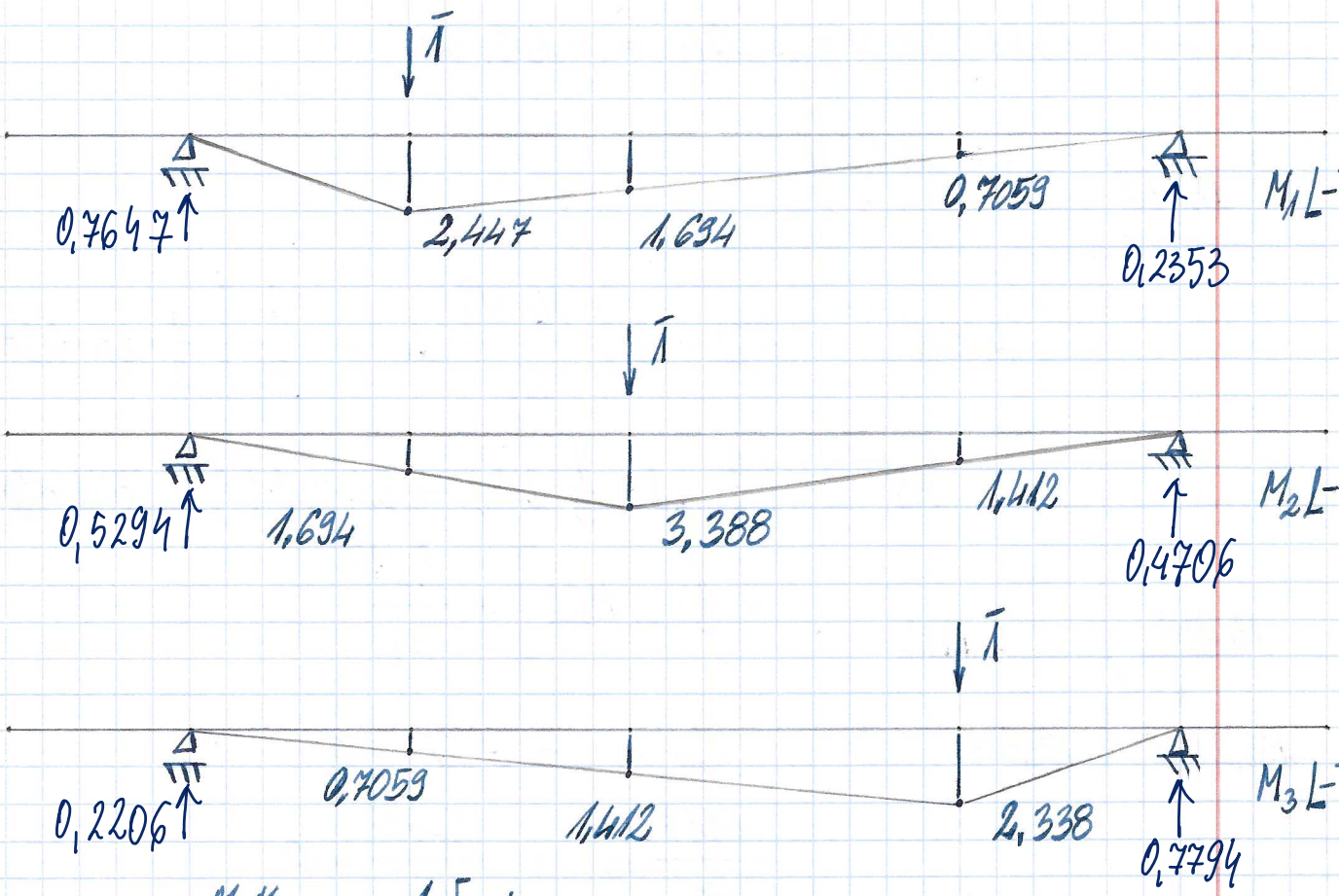
$m_1 = 400\,kg = 8\,m_0$

$m_2 = 450\,kg = 9\,m_0$

$m_3 = 600\,kg = 12\,m_0$

$m_0 = 50\,kg$

$$\begin{cases} q_1(t) = \delta_{11} 8 m_0 a_1 \omega^2 \sin \omega t + \delta_{12} 9 m_0 a_2 \omega^2 \sin \omega t + \delta_{13} 12 m_0 a_3 \omega^2 \sin \omega t \\ q_2(t) = \delta_{21} 8 m_0 a_1 \omega^2 \sin \omega t + \delta_{22} 9 m_0 a_2 \omega^2 \sin \omega t + \delta_{23} 12 m_0 a_3 \omega^2 \sin \omega t \\ q_3(t) = \delta_{31} 8 m_0 a_1 \omega^2 \sin \omega t + \delta_{32} 9 m_0 a_2 \omega^2 \sin \omega t + \delta_{33} 12 m_0 a_3 \omega^2 \sin \omega t \end{cases}$$



$$\delta_{11} = \sum \int \frac{M_1 M_1}{EJ} dx = \frac{1}{EJ}\left[\frac{1}{2}\cdot 3,2\cdot 2,447\cdot\frac{2}{3}\cdot 2,447 + \frac{1}{2}\cdot 10,4\cdot 2,447\cdot\frac{2}{3}\cdot 2,447\right] =$$

$$= \frac{27,15}{EJ}$$

$$\delta_{12} = \delta_{21} = \sum \int \frac{M_1 M_2}{EJ} dx = \frac{1}{EJ}\left[\frac{1}{2}\cdot 3,2\cdot 2,447\cdot\frac{2}{3}\cdot 1,694 + \frac{1}{2}\cdot 7,2\cdot 1,694\cdot\right.$$

$$\cdot\frac{2}{3}\cdot 3,388 + \frac{1}{2}\cdot 3,2\cdot 2,447\cdot\left(\frac{2}{3}\cdot 1,694 + \frac{1}{3}\cdot 3,388\right) + \frac{1}{2}\cdot 3,2\cdot 1,694\cdot$$

$$\left.\cdot\left(\frac{2}{3}\cdot 3,388 + \frac{1}{3}\cdot 1,694\right)\right] = \frac{34,69}{EJ}$$

$$\delta_{13} = \delta_{31} = \sum \int \frac{M_1 M_3}{EJ} dx = \frac{1}{EJ}\left[\frac{1}{2}\cdot 3,2\cdot 2447\cdot\frac{2}{3}\cdot 0,7059 + \frac{1}{2}\cdot 3,0\cdot 0,7059\cdot\right.$$

$$\cdot\frac{2}{3}\cdot 2,338 + \frac{1}{2}\cdot 7,4\cdot 2447\cdot\left(\frac{2}{3}\cdot 0,7059 + \frac{1}{3}\cdot 2,338\right) + \frac{1}{2}\cdot 7,4\cdot 0,7059\cdot$$

$$\left.\cdot\left(\frac{2}{3}\cdot 2,338 + \frac{1}{3}\cdot 0,7059\right)\right] = \frac{19,50}{EJ}$$

$$\delta_{23} = \delta_{32} = \sum \int \frac{M_2 M_3}{EJ} dx = \frac{1}{EJ}\left[\frac{1}{2}\cdot 6,4\cdot 3,388\cdot\frac{2}{3}\cdot 1,412 + \frac{1}{2}\cdot 3,0\cdot 1,412\cdot\right.$$

$$\cdot\frac{2}{3}\cdot 2,338 + \frac{1}{2}\cdot 4,2\cdot 3,388\cdot\left(\frac{2}{3}\cdot 1,412 + \frac{1}{3}\cdot 2,338\right) + \frac{1}{2}\cdot 4,2\cdot 1,412\cdot$$

$$\left.\cdot\left(\frac{2}{3}\cdot 2,338 + \frac{1}{3}\cdot 1,412\right)\right] = \frac{31,77}{EJ}$$

$$\delta_{22} = \sum \int \frac{M_2 M_2}{EJ} dx = \frac{1}{EJ}\left[\frac{1}{2}\cdot 6{,}4 \cdot 3{,}388 \cdot \frac{2}{3}\cdot 3{,}388 + \frac{1}{2}\cdot 7{,}2 \cdot 3{,}388 \cdot \frac{2}{3}\cdot 3{,}388\right] = \frac{52{,}04}{EJ}$$

$$\delta_{33} = \sum \int \frac{M_3 M_3}{EJ} dx = \frac{1}{EJ}\left[\frac{1}{2}\cdot 10{,}6 \cdot 2{,}338 \cdot \frac{2}{3}\cdot 2{,}338 + \frac{1}{2}\cdot 3{,}0 \cdot 2{,}338 \cdot \frac{2}{3}\cdot 2{,}338\right] = \frac{24{,}78}{EJ}$$

$$\begin{cases} a_1 \sin\omega t = \frac{27{,}15}{EJ}\cdot 8 m a_1 \omega^2 \sin\omega t + \frac{34{,}69}{EJ}\cdot 9 m a_2 \omega^2 \sin\omega t + \frac{19{,}50}{EJ}\cdot 12 m a_3 \omega^2 \sin\omega t \\ a_2 \sin\omega t = \frac{34{,}69}{EJ}\cdot 8 m a_1 \omega^2 \sin\omega t + \frac{52{,}04}{EJ}\cdot 9 m a_2 \omega^2 \sin\omega t + \frac{31{,}17}{EJ}\cdot 12 m a_3 \omega^2 \sin\omega t \\ a_3 \sin\omega t = \frac{19{,}50}{EJ}\cdot 8 m a_1 \omega^2 \sin\omega t + \frac{31{,}17}{EJ}\cdot 9 m a_2 \omega^2 \sin\omega t + \frac{24{,}78}{EJ}\cdot 12 m a_3 \omega^2 \sin\omega t \end{cases}$$

$$\lambda = \frac{m\omega^2}{EJ}$$

$$\begin{cases} a_1 = 217{,}2\,\lambda a_1 + 312{,}3\,\lambda a_2 + 234\,\lambda a_3 \\ a_2 = 277{,}5\,\lambda a_1 + 468{,}4\,\lambda a_2 + 381{,}2\,\lambda a_3 \\ a_3 = 156\,\lambda a_1 + 285{,}9\,\lambda a_2 + 297{,}4\,\lambda a_3 \end{cases}$$

$$\begin{cases} 0 = (1 - 217{,}2\lambda) a_1 - 312{,}3\,\lambda a_2 - 234\,\lambda a_3 \\ 0 = -277{,}5\,\lambda a_1 + (1 - 468{,}4\lambda) a_2 - 381{,}2\lambda a_3 \\ 0 = -156\,\lambda a_1 - 285{,}9\,\lambda a_2 + (1 - 297{,}4\lambda) a_3 \end{cases}$$

$$1 - 982{,}98\lambda + 73486\lambda^2 - 847728\lambda^3 = 0$$

$$\lambda_1 = 0{,}001107$$

$$\lambda_2 = 0{,}01511$$

$$\lambda_3 = 0{,}07047$$

$$\lambda = \frac{m\omega^2}{EJ} \qquad \frac{\lambda EJ}{m} = \omega^2$$

$$\omega = \sqrt{\frac{\lambda \cdot EJ}{m}}$$

Przekrój I 100 PE $J_y = 171\ cm^4$ $E = 210\ GPa$

$EJ = 210\ GPa \cdot 171\ cm^4 = 359\,100\ Nm^2$

$$\omega_1 = \sqrt{\frac{0{,}001107 \cdot 359\,100}{50}} = 2{,}82\ \frac{rad}{s} \qquad \frac{P}{\omega_1} = 528$$

$$\omega_2 = \sqrt{\frac{0{,}01511 \cdot 359\,100}{50}} = 10{,}417\ \frac{rad}{s} \qquad \frac{P}{\omega_1} = 14{,}3$$

$$\omega_3 = \sqrt{\frac{0{,}07047 \cdot 359\,100}{50}} = 22{,}4697\ \frac{rad}{s} \qquad \frac{P}{\omega_1} = 6{,}62$$

$P = 23{,}7\ Hz = 23{,}7 \cdot 2\pi = 148{,}9\ \frac{rad}{s}$

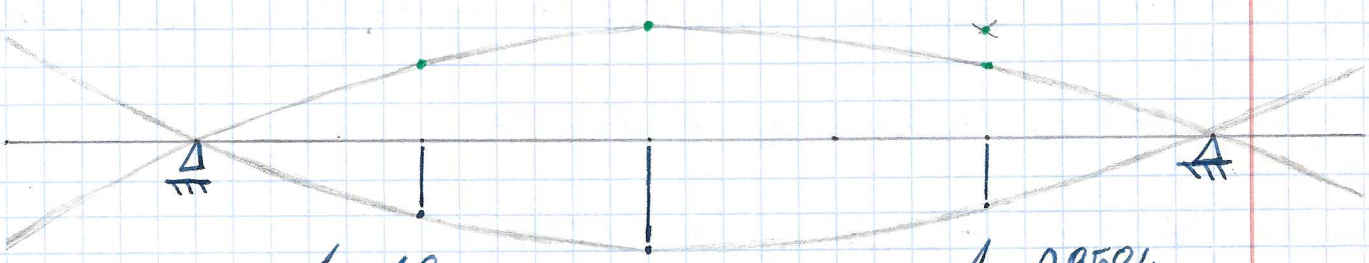
Wniosek: Układ znajduje się poza strefą rezonansową

Postacie drgań własnych:

I postać $\lambda_1 = 0{,}001107$ $a_1^1 = 1{,}0$

$$\begin{cases} (1 - 217{,}2 \cdot 0{,}001107) \cdot 1{,}0 - 312{,}3 \cdot 0{,}001107 \cdot a_2^1 - 234 \cdot 0{,}001107 \cdot a_3^1 = 0 \\ -277{,}5 \cdot 0{,}001107 \cdot 1{,}0 + (1 - 468{,}4 \cdot 0{,}001107) \cdot a_2^1 - 381{,}2 \cdot 0{,}001107\, a_3^1 = 0 \\ -156 \cdot 0{,}001107 \cdot 1{,}0 - 285{,}3 \cdot 0{,}001107 \cdot a_2^1 + (1 - 297{,}4 \cdot 0{,}001107) \cdot a_3^1 = 0 \end{cases}$$

$a_2^1 = 1{,}4787$ $a_3^1 = 0{,}95840$

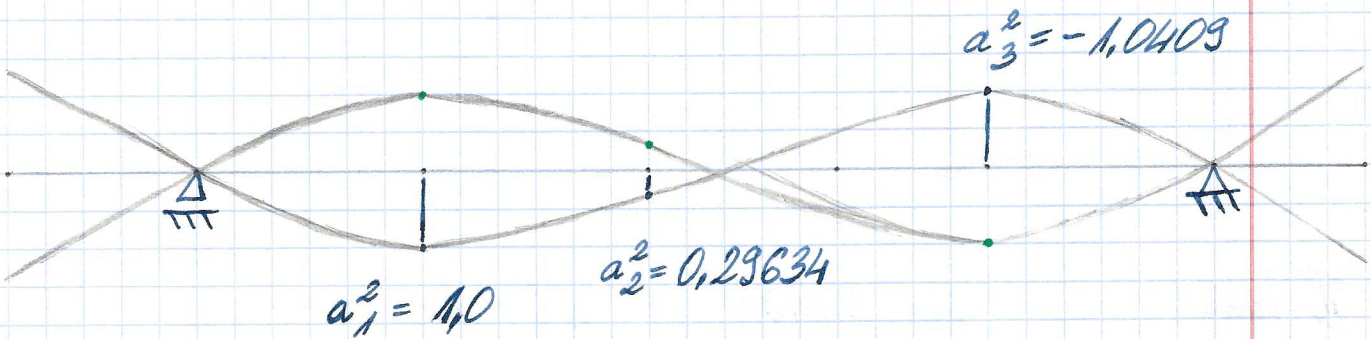


$a_1^1 = 1{,}0$ $a_2^1 = 1{,}4787$ $a_3^1 = 0{,}9584$

II postać $\lambda_2 = 0{,}01511$ $a_1^2 = 1{,}0$

$$\begin{cases} (1 - 217{,}2 \cdot 0{,}01511) \cdot 1{,}0 - 312{,}3 \cdot 0{,}01511 \cdot a_2^2 - 234 \cdot 0{,}01511 \cdot a_3^2 = 0 \\ -277{,}5 \cdot 0{,}01511 \cdot 1{,}0 + (1 - 468{,}4 \cdot 0{,}01511) \cdot a_2^2 - 381{,}2 \cdot 0{,}01511 \cdot a_3^2 = 0 \\ -156{,}0 \cdot 0{,}01511 \cdot 1{,}0 - 285{,}9 \cdot 0{,}01511 \cdot a_2^2 + (1 - 297{,}4 \cdot 0{,}01511) \cdot a_3^2 = 0 \end{cases}$$

$a_2^2 = 0{,}29634$ $a_3^2 = -1{,}0409$



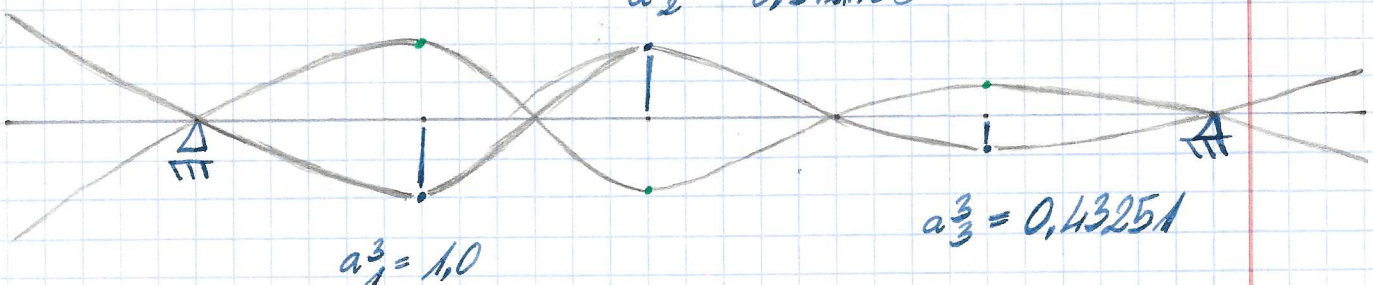
III postać $\lambda_3 = 0{,}07047$ $a_1^3 = 1{,}0$

$$(1 - 217{,}2 \cdot 0{,}07047) \cdot 1{,}0 - 312{,}3 \cdot 0{,}07047 \cdot a_2^3 - 234{,}0 \cdot 0{,}07047 \cdot a_3^3 = 0$$

$$-277{,}5 \cdot 0{,}07047 \cdot 1{,}0 + (1 - 468{,}4 \cdot 0{,}07047) \cdot a_2^3 - 381{,}2 \cdot 0{,}07047 \cdot a_3^3 = 0$$

$$-156 \cdot 0{,}07047 \cdot 1{,}0 - 285{,}9 \cdot 0{,}07047 \cdot a_2^3 + (1 - 297{,}4 \cdot 0{,}07047) \cdot a_3^3 = 0$$

$a_2^3 = -0{,}97408$ $a_3^3 = 0{,}43251$



Warunek ortogonalności drgań

I i II postać

$$400 \cdot 1{,}0 \cdot 1{,}0 + 450 \cdot 1{,}4787 \cdot 0{,}29634 + 600 \cdot 0{,}9584 \cdot (-1{,}0409) =$$

$$= -1{,}37$$

$400 + 197{,}2 - 598{,}6 \simeq -1{,}4$

max składnik sumy $\cdot$ 598,6

$\frac{1{,}4}{598{,}6} \cdot 100\% = 0{,}23\% \Rightarrow$ w tym przypadku $1{,}4 \cong 0 \Rightarrow$ ok

II i III postać

$$400 \cdot 1{,}0 \cdot 1{,}0 + 450 \cdot 0{,}29634 \cdot (-0{,}97408) + 600 \cdot (-1{,}0409) \cdot 0{,}43251 =$$

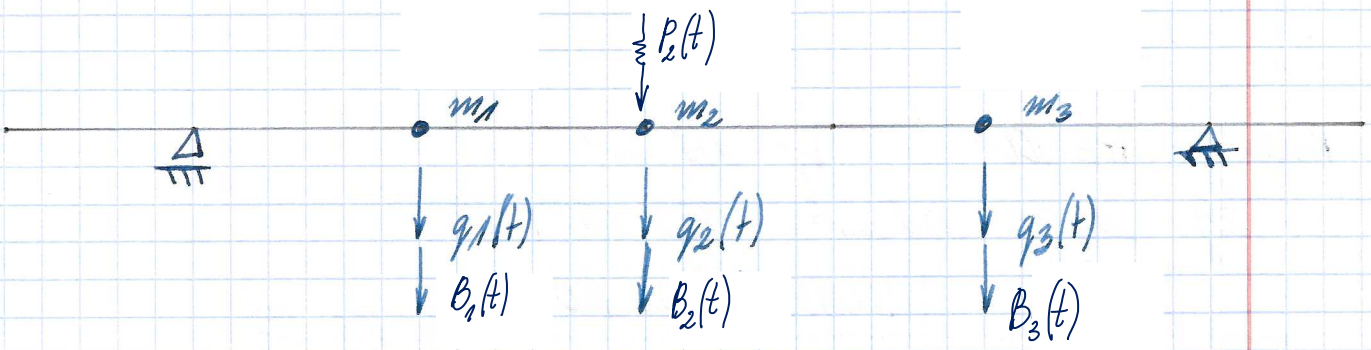
$$= -0{,}016 \qquad \cong 0 \Rightarrow \text{ok}$$

I i III postać

$$400 \cdot 1{,}0 \cdot 1{,}0 + 450 \cdot 1{,}4787 \cdot (-0{,}97408) + 600 \cdot 0{,}95840 \cdot 0{,}43251 =$$

$$= 0{,}54 \qquad \cong 0 \Rightarrow \text{ok}$$

Drgania wymuszone



$$\begin{cases} q_1(t) = \delta_{11} B_1(t) + \delta_{12}\left(B_2(t) + P_2(t)\right) + \delta_{13} B_3(t) \\ q_2(t) = \delta_{21} B_1(t) + \delta_{22}\left(B_2(t) + P_2(t)\right) + \delta_{23} B_3(t) \\ q_3(t) = \delta_{31} B_1(t) + \delta_{32}\left(B_2(t) + P_2(t)\right) + \delta_{33} B_3(t) \end{cases}$$

$$\begin{cases} a_1 \sin pt = \delta_{11} m_1 a_1 p^2 \sin pt + \delta_{12}\left(m_2 a_2 p^2 \sin pt + P \sin pt\right) + \delta_{13} m_3 a_3 p^2 \sin pt \\ a_2 \sin pt = \delta_{21} m_1 a_1 p^2 \sin pt + \delta_{22}\left(m_2 a_2 p^2 \sin pt + P \sin pt\right) + \delta_{23} m_3 a_3 p^2 \sin pt \\ a_3 \sin pt = \delta_{31} m_1 a_1 p^2 \sin pt + \delta_{32}\left(m_2 a_2 p^2 \sin pt + P \sin pt\right) + \delta_{33} m_3 a_3 p^2 \sin pt \end{cases}$$

$$\begin{cases} a_1 = \dfrac{27{,}15}{EJ} \cdot m_1 a_1 p^2 + \dfrac{34{,}69}{EJ} \cdot \left(m_2 a_2 p^2 + P\right) + \dfrac{19{,}50}{EJ} \cdot m_3 a_3 p^2 \\ a_2 = \dfrac{34{,}69}{EJ} \cdot m_1 a_1 p^2 + \dfrac{52{,}04}{EJ} \cdot \left(m_2 a_2 p^2 + P\right) + \dfrac{31{,}77}{EJ} \cdot m_3 a_3 p^2 \\ a_3 = \dfrac{19{,}50}{EJ} \cdot m_1 a_1 p^2 + \dfrac{31{,}77}{EJ} \cdot \left(m_2 a_2 p^2 + P\right) + \dfrac{24{,}78}{EJ} \cdot m_3 a_3 p^2 \end{cases}$$

$$\begin{cases} a_1 = 670{,}5\,a_1 + 364{,}1\,a_2 + 0{,}3382 + 722{,}4\,a_3 \\ a_2 = 857{,}0\,a_1 + 1446\,a_2 + 0{,}5072 + 1177\,a_3 \\ a_3 = 481{,}6\,a_1 + 882{,}7\,a_2 + 0{,}3096 + 918{,}3\,a_3 \end{cases}$$

$$\begin{cases} 669{,}5\,a_1 + 364{,}1\,a_2 + 722{,}4\,a_3 = -0{,}3382 \\ 857{,}0\,a_1 + 1446\,a_2 + 1177\,a_3 = -0{,}5072 \\ 481{,}6\,a_1 + 882{,}7\,a_2 + 918{,}3\,a_3 = -0{,}3096 \end{cases}$$

$a_1 = 3{,}5690 \cdot 10^{-6}\ m$

$a_2 = -3{,}5460 \cdot 10^{-4}\ m$

$a_3 = 1{,}8320 \cdot 10^{-6}\ m$

Dla $\sin pt = 1{,}0$

$B_1(t) = m_1 a_1 p^2 \sin pt = 400 \cdot 3{,}569 \cdot 10^{-6} \cdot 148{,}9^2 = 31{,}66\ N$

$B_2(t) = m_2 a_2 p^2 \sin pt = 450 \cdot (-3{,}546 \cdot 10^{-4}) \cdot 148{,}9^2 = -3538\ N$

$B_3(t) = m_3 a_3 p^2 \sin pt = 600 \cdot 1{,}832 \cdot 10^{-6} \cdot 148{,}9^2 = 24{,}37\ N$

$B_2(t) + P_2(t) = -3538 + 3500 = -38\ N$

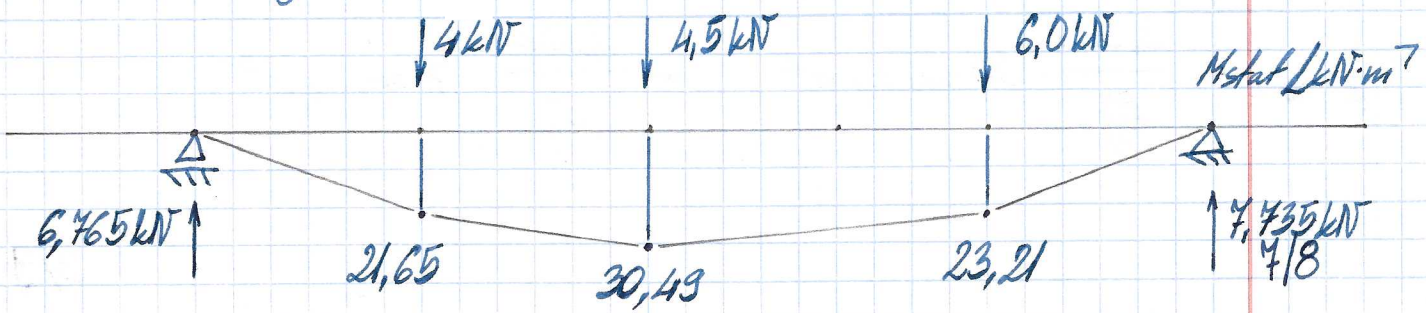
Obciążenie statyczne siłami ciężkości

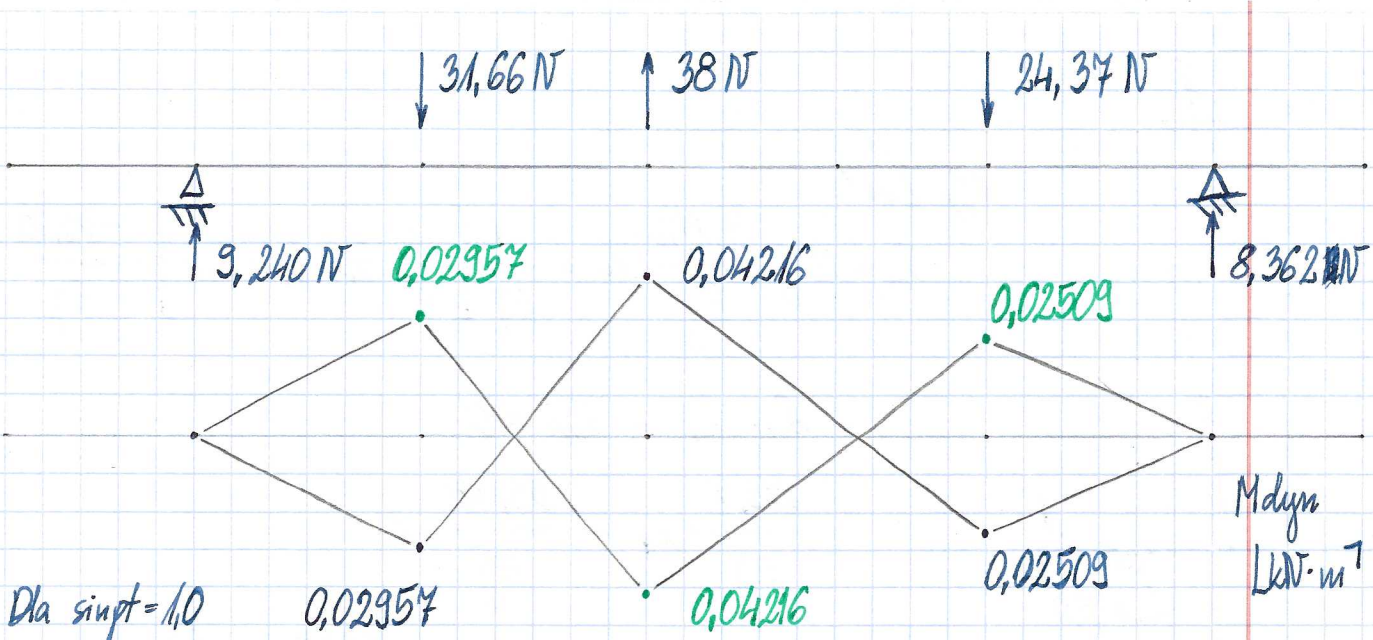
$F_c = m \cdot g \qquad g = 10\ \frac{m}{s^2}$

$F_{c1} = m_1 \cdot g = 400 \cdot 10 = 4000\ N = 4{,}0\ kN$

$F_{c2} = m_2 \cdot g = 450 \cdot 10 = 4500\ N = 4{,}5\ kN$

$F_{c3} = m_3 \cdot g = 600 \cdot 10 = 6000\ N = 6{,}0\ kN$





Dla sinpt = 1,0 0,02957 0,04216

Dla sinpt = −1,0

Maksymalne momenty

$M_A = 1,2 \cdot 0,0 + 5,0 \cdot 0,0 = 0,0$ kN·m

$M_B = 1,2 \cdot 0,0 + 5,0 \cdot 0,0 = 0,0$ kN·m

$M_C = 1,2 \cdot 21,65 + 5,0 \cdot 0,02957 = 26,12$ kN·m

$M_D = 1,2 \cdot 30,49 + 5,0 \cdot 0,04216 = 36,80$ kN·m

$M_E = 1,2 \cdot 23,21 + 5,0 \cdot 0,02509 = 27,97$ kN·m

$M_G = 1,2 \cdot 0,0 + 5,0 \cdot 0,0 = 0,0$ kN·m

$M_H = 1,2 \cdot 0,0 + 5,0 \cdot 0,0 = 0,0$ kN·m

Maksymalne naprężenia

$$\sigma_x = \frac{M_{max}}{W_y}$$

Dla IPE 100

$W_y = 34,20\ cm^3$

$$\sigma_x = \frac{3680\ kN \cdot cm}{34,20\ cm^3} = 107,6\ \frac{kN}{cm^2} = 1076\ MPa$$

Wniosek:

Naprężenia maksymalne przekroczone.

		L1		L2		L3		L4			
Dł elementów		3,2000		3,2000		4,2000		3,0000		Va	Vb
M1	0,0000		2,4471		1,6941		0,7059		0,0000	0,7647	0,2353
M2	0,0000		1,6941		3,3882		1,4118		0,0000	0,5294	0,4706
M3	0,0000		0,7059		1,4118		2,3382		0,0000	0,2206	0,7794

11	27,1460	0,0000	6,3873	8,5983	5,2724	4,8551	1,5347	0,4983	0,0000
12	34,6955	0,0000	4,4220	8,8440	7,6534	9,7103	3,0693	0,9965	0,0000
13	19,4965	0,0000	1,8425	3,6850	3,1889	6,1212	3,0083	1,6505	0,0000
22	52,0433	0,0000	3,0614	6,1227	15,3069	19,4206	6,1387	1,9931	0,0000
23	31,7647	0,0000	1,2756	2,5511	6,3779	12,2425	6,0166	3,3010	0,0000
33	24,7853	0,0000	0,5315	1,0630	2,6574	5,1010	9,9650	5,4673	0,0000

Drgania własne

m1	400,0000		P1dyn		m1a1w^2cospt					
m2	450,0000		P2dyn		m2a2w^2cospt					
m3	600,0000		P3dyn		m3a3w^2cospt			α = m*w^2/EI		
m0	50,0000		P4dyn		0,0000					
	[cm^4]		[Gpa]		[N*m^2]		[Hz]	[rad/s]		
I	171,0000	E	210,0000	EI	359100,0000	p	23,7000	148,9115		
a1	=	217,1683	αa1	312,2598	αa2	233,9576	αa3			
a2	=	277,5642	αa1	468,3896	αa2	381,1765	αa3			
a3	=	155,9718	αa1	285,8824	αa2	297,4235	αa3			
det	=	1,0000	x^0	-982,981490	x^1	73485,90195	x^2	-847728,0405	x^3	

lambda^3*	-847728,0405	a		
lambda^2*	73485,90195	b		
lambda*	-982,981490	c		
wyr wolny	1	d		
f	-0,001345256			
g	-1,59255E-05			
h	-2,67625E-11		wyróżnik	
	9,01676E-11			
i	9,49566E-06			
j	0,021175895			
	0,838565334			
k	0,576151842			
m	0,981614894			
n	0,330600362			
p	0,028895235			
lambda1	0,0011079			
lambda2	0,01510942			
lambda3	0,07046838			

α1	=	0,00110790	0,0000			
α2	=	0,01510942	0,0000			
α3	=	0,07046838	0,0000			

Dane - analiza symulacji - szukaj wyniku

						Rezonans		
w1	=	2,8208	p/w1	52,7904	NIE	TAK	1,25 < p/w < 0,75	
w2	=	10,4171	p/w2	14,2949	NIE	NIE	p/w > 1,25 lub p/w < 0,75	
w3	=	22,4968	p/w3	6,6192	NIE			

Postacie drgań własnych

I postać		0,0011079		a1	=	1,0000								
								x	y	W		x	w	Wx
0,0000	=	0,7594	a1	-0,3460	a2	-0,2592	a3	-0,3460	-0,2592	0,2708		-0,3460	-0,7594	0,2589
0,0000	=	-0,3075	a1	0,4811	a2	-0,4223	a3	0,4811	-0,4223			0,4811	0,3075	
0,0000	=	-0,1728	a1	-0,3167	a2	0,6705	a3							
	a2	=	1,47865		a3	=	0,95622							
II postać		0,015109		a1	=	1,0000								
								x	y	W		x	w	Wx
0,0000	=	-2,2813	a1	-4,7181	a2	-3,5350	a3	-4,7181	-3,5350	5,6907		-4,7181	2,2813	-5,9232
0,0000	=	-4,1938	a1	-6,0771	a2	-5,7594	a3	-6,0771	-5,7594			-6,0771	4,1938	
0,0000	=	-2,3566	a1	-4,3195	a2	-3,4939	a3							
	a2	=	0,29633		a3	=	-1,0409							
III postać		0,070468		a1	=	1,0000								
								x	y	W		x	w	Wx
0,0000	=	-14,3035	a1	-22,0044	a2	-16,4866	a3	-22,0044	-16,4866	63,3773		-22,0044	14,3035	27,4114
0,0000	=	-19,5595	a1	-32,0067	a2	-26,8609	a3	-32,0067	-26,8609			-32,0067	19,5595	
0,0000	=	-10,9911	a1	-20,1457	a2	-19,9590	a3							
	a2	=	-0,97408		a3	=	0,43251							

Warunek ortogonalności drgań

I i II postać	0,000000
I i III postać	0,000000
II i III postać	0,000000

Drgania wymuszone

m1	400,0000		P1dyn		m1a1w^2cospt				
m2	450,0000		P2dyn		m2a2w^2cospt				
m3	600,0000		P3dyn		m3a3w^2cospt				
P (t)	3500,0000		P4dyn		P (t)				
	[cm^4]		[Gpa]		[N*m^2]		[Hz]	[rad/s]	
I	171,0000	E	210,0000	EI	359100,0000	p	23,7000	148,9115	
									M2
a1	=	670,5134	a1	964,1110	a2	722,3510	a3		0,3381631
a2	=	856,9876	a1	1446,1666	a2	1176,8934	a3		0,5072446
a3	=	481,5673	a1	882,6700	a2	918,3037	a3		0,3095975
0,0000	=	669,5134	αa1	964,1110	αa2	722,3510	αa3		0,3381631
0,0000	=	856,9876	αa1	1445,1666	αa2	1176,8934	αa3		0,5072446
0,0000	=	481,5673	αa1	882,6700	αa2	917,3037	αa3		0,3095975

a1	=	0,0000035690
a2	=	- 0,0003546021
a3	=	0,0000018320

					Fc	g	=	10,0000 m/s^2
P1dyn	m1a1w^2cospt	P(t)	31,6562	0,0317	4,0000			
P2dyn	m2a2w^2cospt		-38,4274	-0,0384	4,5000			
P3dyn	m3a3w^2cospt		24,3737	0,0244	6,0000			
P4dyn	0,0000							

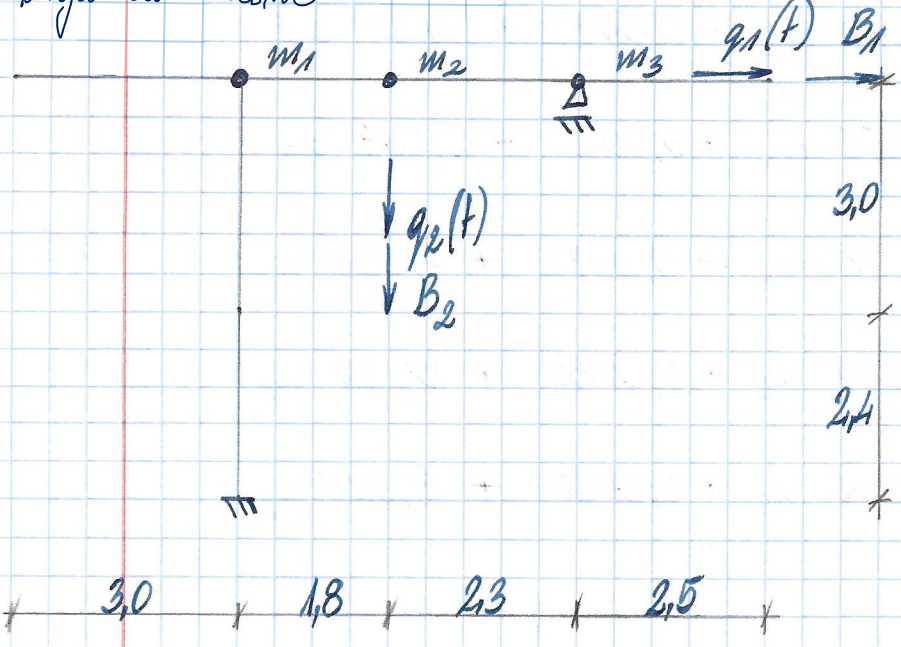
			L1		L2		L3		L4			
Dł elementów			3,2000		3,2000		4,2000		3,0000		Va	Vb
Mdyn		0,0000		0,02957		-0,04216		0,02509		0,0000	0,009240	0,008362
Mstat		0,0000		21,6471		30,4941		23,2059		0,0000	6,7647	7,7353
wsp dyn	5	0		26,12431516		36,80375056		27,97249156		0		
wsp stat	1,2											

Naprężenia maksymalne

Mmax	36,80375056	kN*m	Wy	34,2	cm3
σmax	107,6133057	kN/cm2			
	1076,133057	Mpa			

Dynamika – ujęcie klasyczne – Rama

Drgania własne



$m_0 = 50\ kg$

$m_1 = 400\ kg = 8\ m_0$

$m_2 = 350\ kg = 7\ m_0$

$m_3 = 950\ kg = 19\ m_0$

$q_i(t) = a_i \sin \omega t$

$P_i(t) = -m_i \ddot{q}_i(t)$

$\ddot{q}_i(t) = -a_i \omega^2 \sin \omega t$

$B_1(t) = -(m_1 + m_2 + m_3) \cdot \ddot{q}_1(t)$

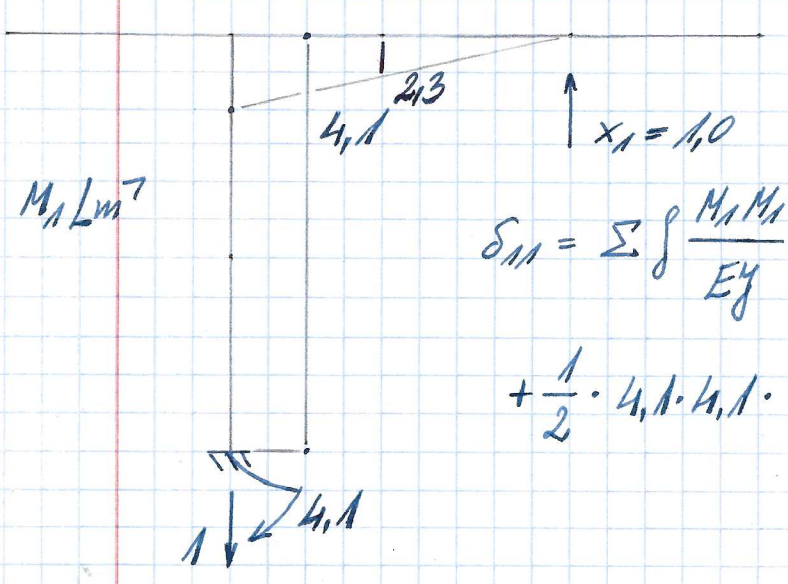
$B_2(t) = -m_2 \cdot \ddot{q}_2(t)$

$$\begin{cases} q_1(t) = \delta_{11} B_1 + \delta_{12} B_2 \\ q_2(t) = \delta_{21} B_1 + \delta_{22} B_2 \end{cases}$$

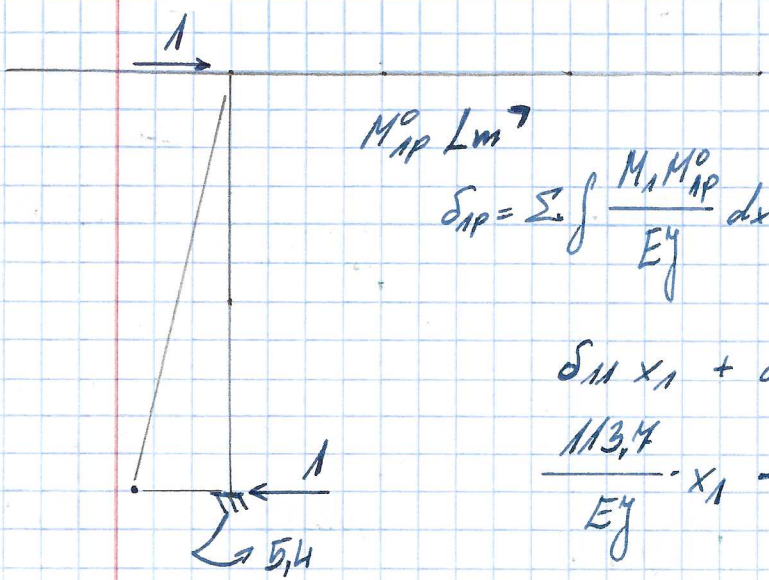
$$\begin{cases} q_1(t) = \delta_{11} \cdot (8 m_0 + 7 m_0 + 19 m_0)\, a_1 \omega^2 \sin \omega t + \delta_{12} \cdot 7 m_0 a_2 \omega^2 \sin \omega t \\ q_2(t) = \delta_{21} \cdot (8 m_0 + 7 m_0 + 19 m_0)\, a_1 \omega^2 \sin \omega t + \delta_{22} \cdot 7 m_0 a_2 \omega^2 \sin \omega t \end{cases}$$

$$\begin{cases} a_1 \sin \omega t = \delta_{11} \cdot 34 m_0 a_1 \omega^2 \sin \omega t + \delta_{21} \cdot 7 m_0 a_2 \omega^2 \sin \omega t \\ a_2 \sin \omega t = \delta_{21} \cdot 34 m_0 a_1 \omega^2 \sin \omega t + \delta_{22} \cdot 7 m_0 a_2 \omega^2 \sin \omega t \end{cases}$$

$$\lambda = \frac{m \omega^2}{EJ}$$



$$\delta_{11} = \sum \int \frac{M_1 M_1}{EJ} dx = \frac{1}{EJ} \left[5,4 \cdot 4,1 \cdot 4,1 + \frac{1}{2} \cdot 4,1 \cdot 4,1 \cdot \frac{2}{3} \cdot 4,1 \right] = \frac{113,7}{EJ}$$



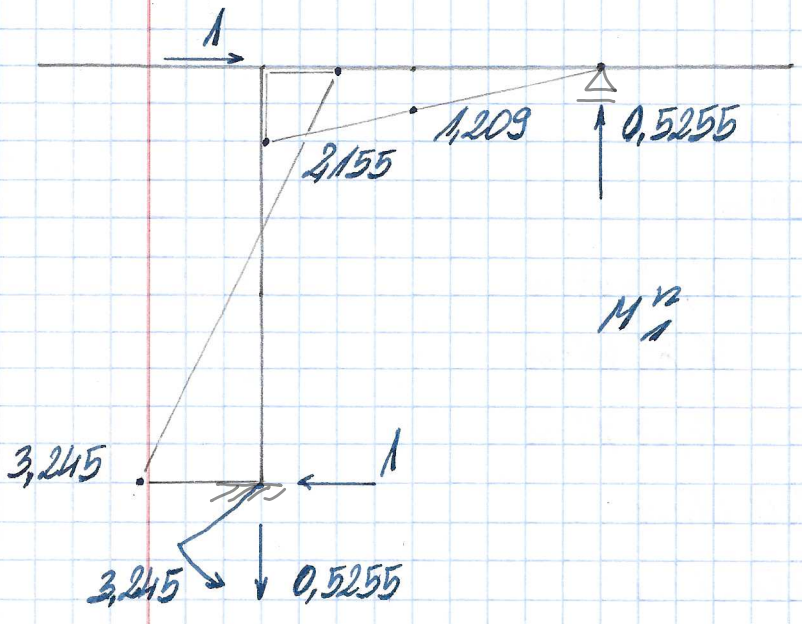
M^{o}_{1p} [m]

$$\delta_{1p} = \sum \int \frac{M_1 M^{o}_{1p}}{EJ} dx = \frac{1}{EJ}\left[-\frac{1}{2}\cdot 5{,}4\cdot 5{,}4\cdot 4{,}1\right] = \frac{-59{,}78}{EJ}$$

$$\delta_{11} x_1 + \delta_{1p} = 0$$

$$\frac{113{,}7}{EJ}\cdot x_1 - \frac{59{,}78}{EJ} = 0$$

$$x_1 = \frac{59{,}78}{EJ}\cdot\frac{EJ}{113{,}7} = 0{,}5255$$

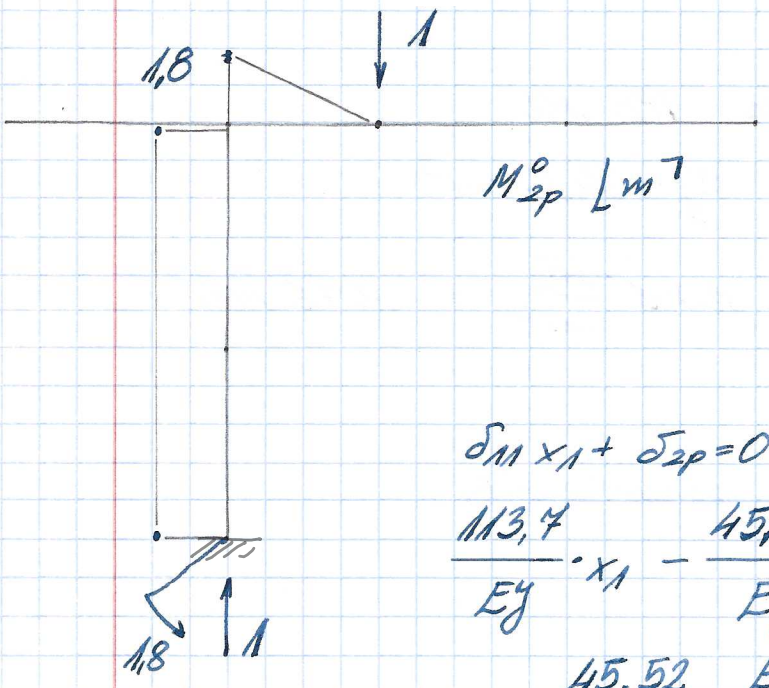


$M^{x_1}_{1}$

$$\delta_{2p} = \sum \int \frac{M_1 M^{o}_{2p}}{EJ} dx =$$

$$= \frac{1}{EJ}\left[-5{,}4\cdot 1{,}8\cdot 4{,}1 - \frac{1}{2}\cdot 1{,}8\cdot 1{,}8\cdot\left(\frac{2}{3}\cdot 4{,}1 + \frac{1}{3}\cdot 2{,}3\right)\right] =$$

$$= \frac{-45{,}52}{EJ}$$

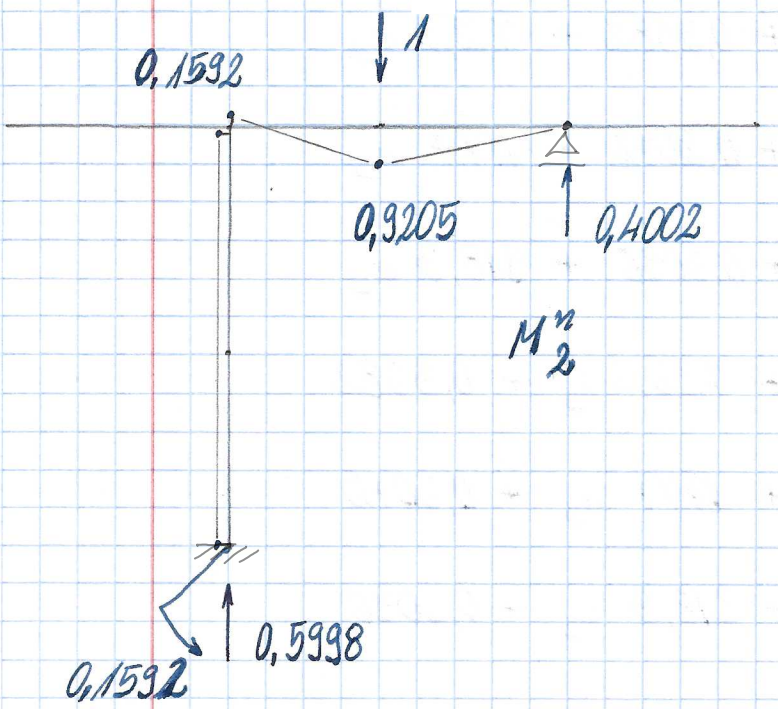


M^{o}_{2p} [m]

$$\delta_{11} x_1 + \delta_{2p} = 0$$

$$\frac{113{,}7}{EJ}\cdot x_1 - \frac{45{,}52}{EJ} = 0$$

$$x_1 = \frac{45{,}52}{EJ}\cdot\frac{EJ}{113{,}7} = 0{,}4002$$



M_2^n

$$\delta_{11}^n = \sum \int \frac{M_1^n M_1^n}{EJ} dx =$$

$$= \frac{1}{EJ}\left[\frac{1}{2}\cdot 4,1\cdot 2,155\cdot \frac{2}{3}\cdot 2,155 \right.$$

$$+ \frac{1}{2}\cdot 5,4\cdot 2,155\cdot\left(\frac{2}{3}\cdot 2,155 - \frac{1}{3}\cdot 3,245\right)$$

$$\left. + \frac{1}{2}\cdot 5,4\cdot 3,245\left(\frac{2}{3}\cdot 3,245 - \frac{1}{3}\cdot 2,155\right)\right]$$

$$= \frac{21,07}{EJ}$$

$$\delta_{12}^n = \delta_{21}^n = \sum \int \frac{M_1^n M_2^n}{EJ} dx = \frac{1}{EJ}\left[5,4\cdot 0,1592\cdot\left(\frac{1}{2}\cdot 3,245 - \frac{1}{2}\cdot 2,155\right)\right.$$

$$+ \frac{1}{2}\cdot 2,3\cdot 0,9205\cdot\frac{2}{3}\cdot 1,209 + \frac{1}{2}\cdot 1,8\cdot 0,9205\cdot\left(\frac{2}{3}\cdot 1,209 + \frac{1}{3}\cdot 2,155\right)$$

$$\left. - \frac{1}{2}\cdot 1,8\cdot 0,1592\cdot\left(\frac{2}{3}\cdot 2,155 + \frac{1}{3}\cdot 1,209\right)\right] = \frac{2,321}{EJ}$$

$$\delta_{22}^n = \sum \int \frac{M_2^n M_2^n}{EJ} dx = \frac{1}{EJ}\left[5,4\cdot 0,1592\cdot 0,1592 + \frac{1}{2}\cdot 2,3\cdot 0,9205\cdot\frac{2}{3}\cdot 0,9205\right.$$

$$+ \frac{1}{2}\cdot 1,8\cdot 0,9205\cdot\left(\frac{2}{3}\cdot 0,9205 - \frac{1}{3}\cdot 0,1592\right) + \frac{1}{2}\cdot 1,8\cdot 0,1592\cdot\left(\frac{2}{3}\cdot 0,1592 -\right.$$

$$\left.\left. - \frac{1}{3}\cdot 0,9205\right)\right] = \frac{1,222}{EJ}$$

$$\begin{cases} a_1 = \dfrac{21,07}{EJ}\cdot 34\, m_0 a_1 \omega^2 + \dfrac{2,321}{EJ}\cdot 7\, m_0 a_2 \omega^2 \\ a_2 = \dfrac{2,321}{EJ}\cdot 34\, m_0 a_1 \omega^2 + \dfrac{1,222}{EJ}\cdot 7\, m_0 a_2 \omega^2 \end{cases}$$

$$\begin{cases} a_1 = 716,5 \lambda a_1 + 16,25 \lambda a_2 \\ a_2 = 78,91 \lambda a_1 + 8,554 \lambda a_2 \end{cases}$$

$$\begin{cases} 0 = (1 - 716,5\lambda)\, a_1 - 16,25\lambda\, a_2 \\ 0 = -78,91\lambda\, a_1 + (1 - 8,554\lambda)\, a_2 \end{cases}$$

$$\det = 1 - 725,0\,\lambda + 4847\,\lambda^2$$

$$\lambda_1 = 0,001392$$

$$\lambda_2 = 0,1482$$

Przekrój: IPE 100 $\quad J_y = 171\ cm^4$

$$EJ = 210\ GPa \cdot 171\ cm^4 = 359\,100\ Nm^2$$

$$\omega_1 = \sqrt{\frac{\lambda_1 \cdot EJ}{m_0}} = \sqrt{\frac{0,001392 \cdot 359\,100}{50}} = 3,162\ \frac{rad}{s}$$

$$\omega_2 = \sqrt{\frac{\lambda_2 \cdot EJ}{m_0}} = \sqrt{\frac{0,1482 \cdot 359\,100}{50}} = 32,62\ \frac{rad}{s}$$

$$p = 24,5\ Hz = 24,5 \cdot 2\pi = 153,9\ \frac{rad}{s}$$

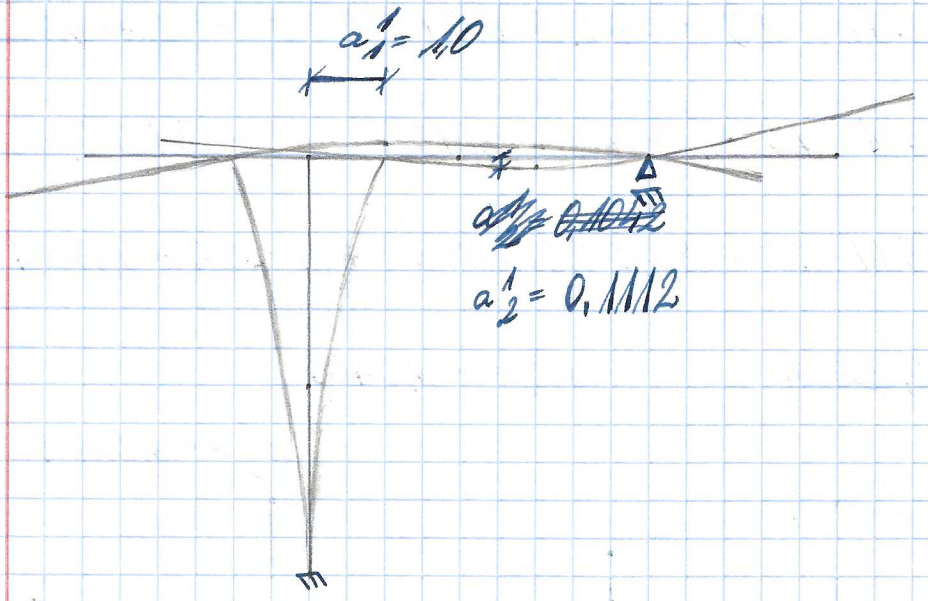
$$\frac{p}{\omega_1} = \frac{153,9}{3,162} = 48,67 \qquad \frac{p}{\omega_2} = \frac{153,9}{32,62} = 4,718$$

Wniosek: Układ znajduje się poza strefą rezonansową.

I postać $\quad \lambda_1 = 0,001392 \quad a_1^1 = 1,0$

$$\begin{cases} 0 = (1 - 716,5 \cdot 0,001392) \cdot 1,0 - 16,25 \cdot 0,001392 \cdot a_2^1 \\ 0 = -78,91 \cdot 0,001392 \cdot 1,0 + (1 - 8,554 \cdot 0,001392) \cdot a_2^1 \end{cases}$$

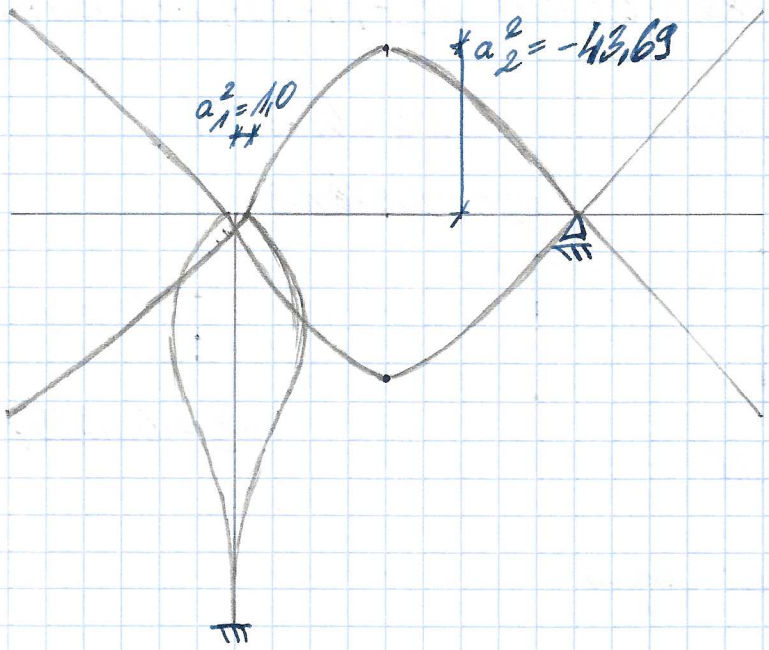
$$a_2^1 = ~~0,1012~~\ 0,1112$$



II postać $\quad \lambda_2 = 0{,}1482 \quad a_1^2 = 1{,}0$

$$\begin{cases} 0 = (1 - 716{,}5 \cdot 0{,}1482) \cdot 1{,}0 - 16{,}25 \cdot 0{,}1482 \cdot a_2^2 \\ 0 = -78{,}91 \cdot 0{,}1482 \cdot 1{,}0 + (1 - 8{,}554 \cdot 0{,}1482) \cdot a_2^2 \end{cases}$$

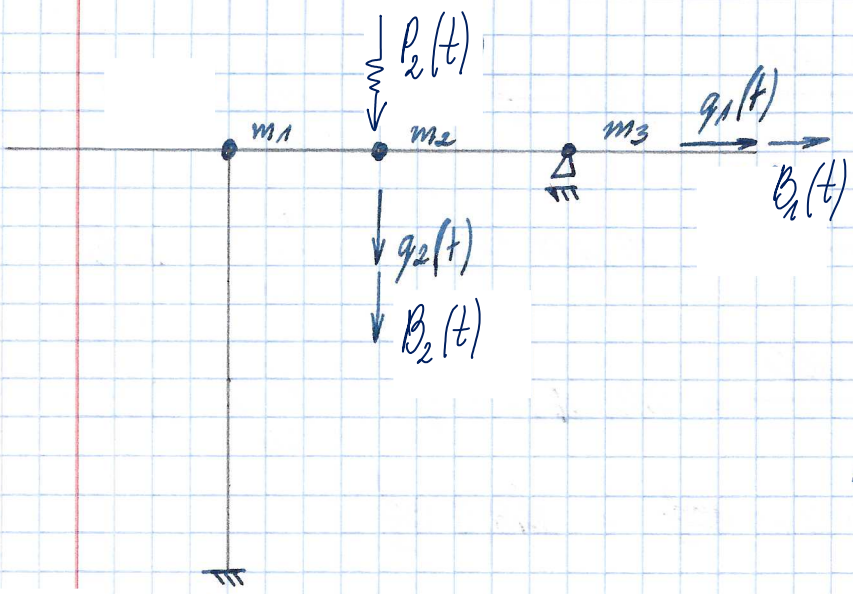
$$a_2^2 = -43{,}69$$



Warunek ortogonalności drgań

$$(400 + 350 + 950) \cdot 1 \cdot 1 + 350 \cdot 0{,}1112 \cdot (-43{,}69) = -0{,}41 \cong 0$$

Drgania wymuszone



$P_2(t) = P\cos pt$

$P = 3400\,N$

$m_1 = 400\,kg$

$m_2 = 350\,kg$

$m_3 = 950\,kg$

$EJ = 359\,100\,Nm^2$

$p = 24,5\,l_2 = 24,5 \cdot 2\pi =$

$= 153,9\,\frac{rad}{s}$

$$B_1(t) = (m_1 + m_2 + m_3)\, a_1 p^2 \cos pt$$

$$B_2(t) = m_2 a_2 p^2 \cos pt$$

$$\begin{cases} q_1(t) = \delta_{11} B_1(t) + \delta_{12}(B_2(t) + P_2(t)) \\ q_2(t) = \delta_{21} B_1(t) + \delta_{22}(B_2(t) + P_2(t)) \end{cases}$$

$$\begin{cases} a_1 \cos pt = \delta_{11}(m_1+m_2+m_3)\cdot a_1 p^2 \cos pt + \delta_{12}(m_2 a_2 p^2 \cos pt + P\cos pt) \\ a_2 \cos pt = \delta_{21}(m_1+m_2+m_3)\cdot a_1 p^2 \cos pt + \delta_{22}(m_2 a_2 p^2 \cos pt + P\cos pt) \end{cases}$$

$$\begin{cases} a_1 = \frac{21,07}{EJ}\cdot(400+350+950)\cdot a_1 \cdot 153,9^2 + \frac{2321}{EJ}\cdot(350\cdot a_2 \cdot 153,9^2 + 3400) \\ a_2 = \frac{2321}{EJ}\cdot(400+350+950)\cdot a_1 \cdot 153,9^2 + \frac{1,122}{EJ}\cdot(350\cdot a_2 \cdot 153,9^2 + 3400) \end{cases}$$

$$\begin{cases} a_1 = 2364\, a_1 + 53,60\, a_2 + 0,02197 \\ a_2 = 260,3\, a_1 + 28,22\, a_2 + 0,01157 \end{cases} \quad \begin{cases} -2363\, a_1 - 53,60 a_2 = 0,02197 \\ -259,3\, a_1 - 28,22 a_2 = 0,01157 \end{cases}$$

$a_1 = 4,362 \cdot 10^{-4}\,m$ $\qquad a_2 = -4,232 \cdot 10^{-4}\,m$

$B_1(t) = (400+350+950)\cdot 4,362\cdot 10^{-4}\cdot 153,9^2 = 17,57\,N$

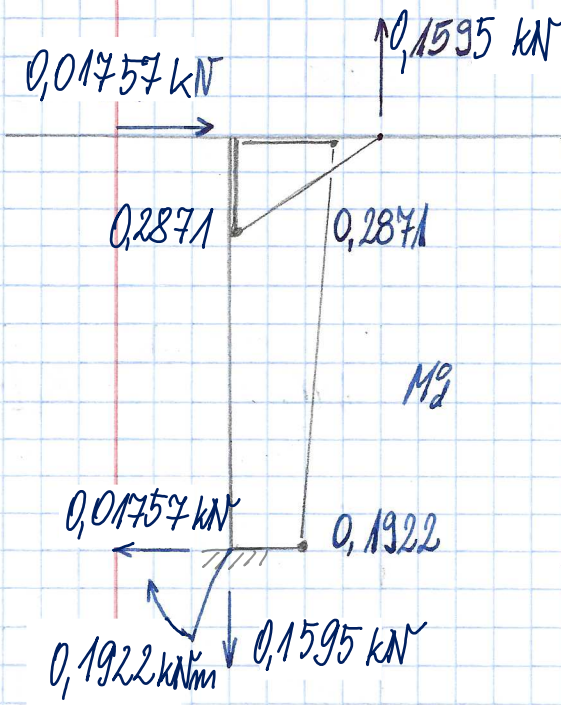
$B_2(t) = 350\cdot(-4,232\cdot 10^{-4})\cdot 153,9^2 = -3559,48\,N$

$B_1^1(t) = 4,133\,N$

$B_1^2(t) = 3,616\,N$

$B_1^3(t) = 9,815\,N$

$B_2(t) + P = -3559,48 \quad + 3400 \quad = -159,5\,kN$



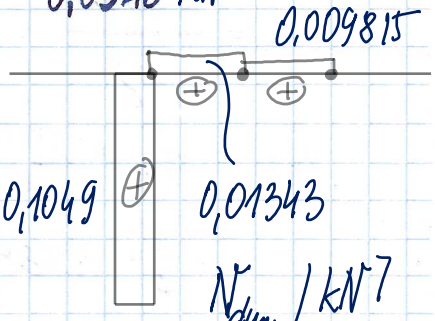
$$\delta^0_{1d} = \sum \int \frac{M_1 M^0_d}{EJ}\,dx = \frac{1}{EJ}\left[\frac{1}{2}\cdot 1,8\cdot 0,2871\cdot\left(\frac{2}{3}\cdot 4,1 + \frac{1}{3}\cdot 2,3\right) + 5,4\cdot 4,1\cdot\left(\frac{1}{2}\cdot 0,2871 + \frac{1}{2}\cdot 0,1922\right)\right] = \frac{6,210}{EJ}$$

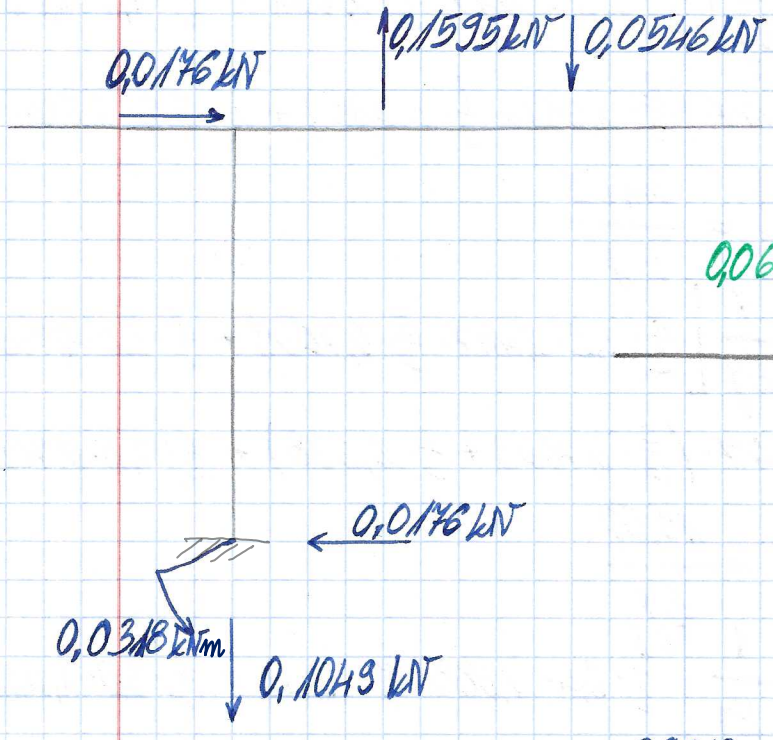
M^0_d

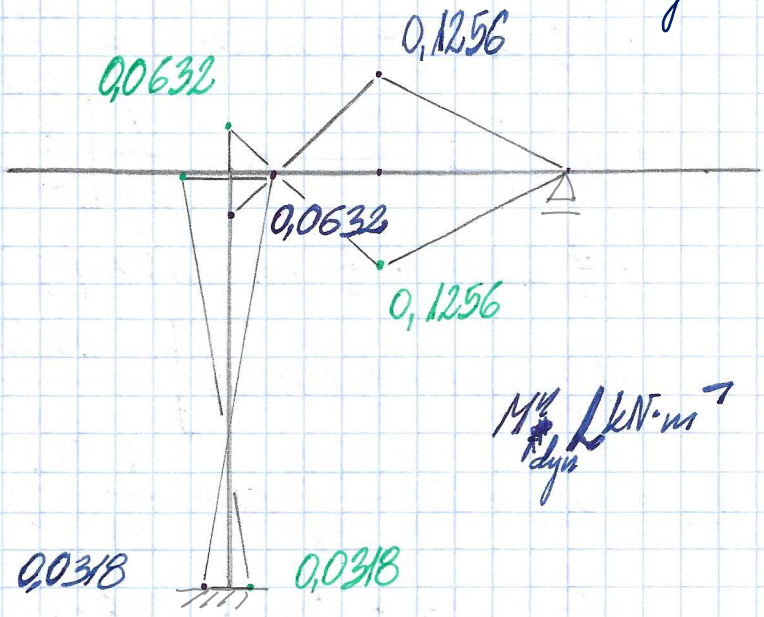
$$\delta_{11}\,x_1 + \delta^0_{1d} = 0$$

$$\frac{113,7}{EJ}\,x_1 + \frac{6,210}{EJ} = 0$$

$$x_1 = -\frac{6,210}{EJ}\cdot\frac{EJ}{113,7} = -0,0546\,kN$$





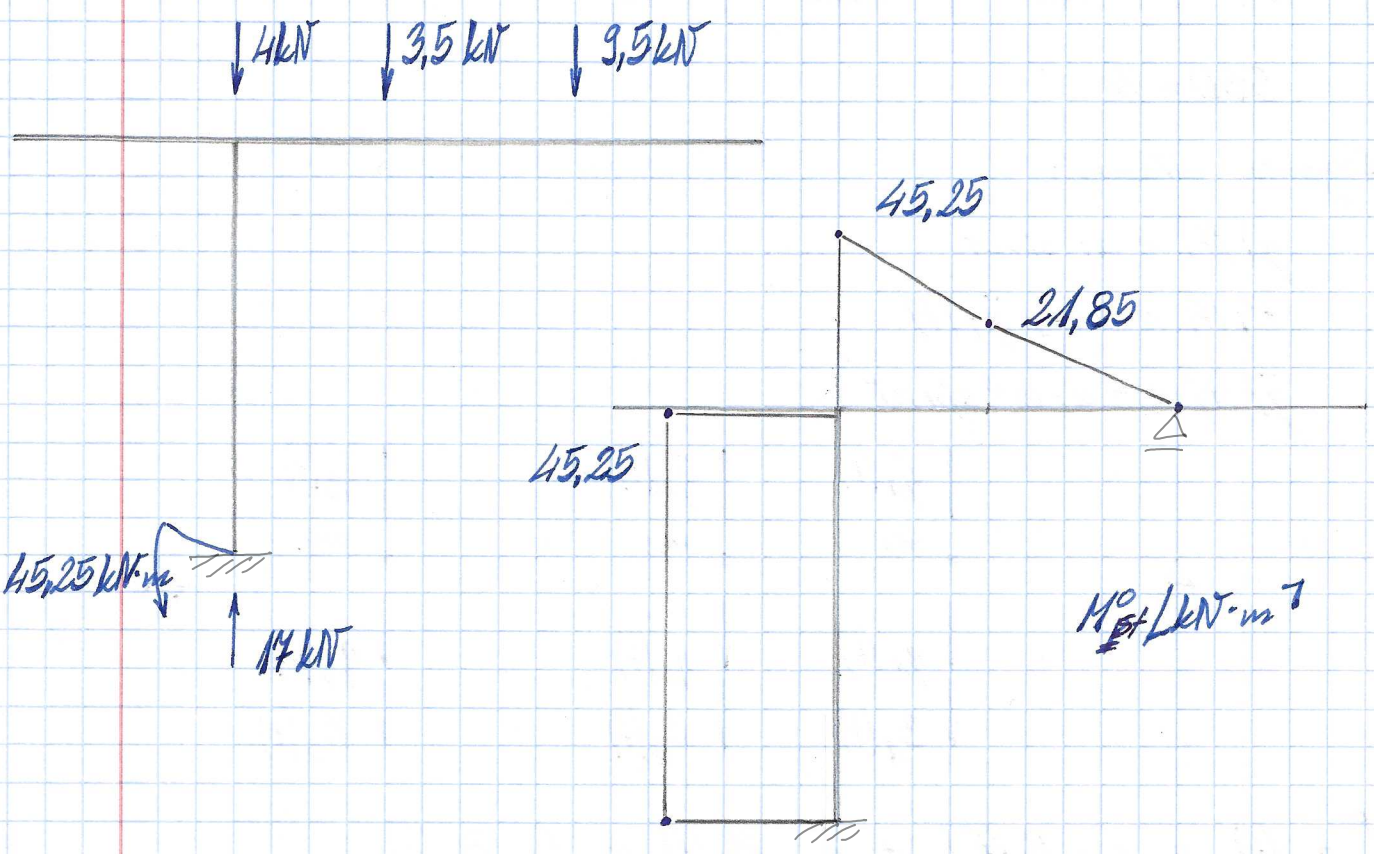


Obciążenie statyczne siłami ciężkości $g = 10 \frac{m}{s^2}$

$F_{c1} = m_1 \cdot g = 400 \cdot 10 = 4000\ N = 4\ kN$

$F_{c2} = m_2 \cdot g = 350 \cdot 10 = 3500\ N = 3{,}5\ kN$

$F_{c3} = m_3 \cdot g = 950 \cdot 10 = 9500\ N = 9{,}5\ kN$



$$\delta^{\circ}_{1st} = \sum \int \frac{M_1 M^{\circ}_{st}}{EJ} dx = \frac{1}{EJ}\Big[-5{,}4 \cdot 4{,}1 \cdot 45{,}25 - \frac{1}{2} \cdot 23 \cdot 23 \cdot \frac{2}{3} \cdot 21{,}85$$

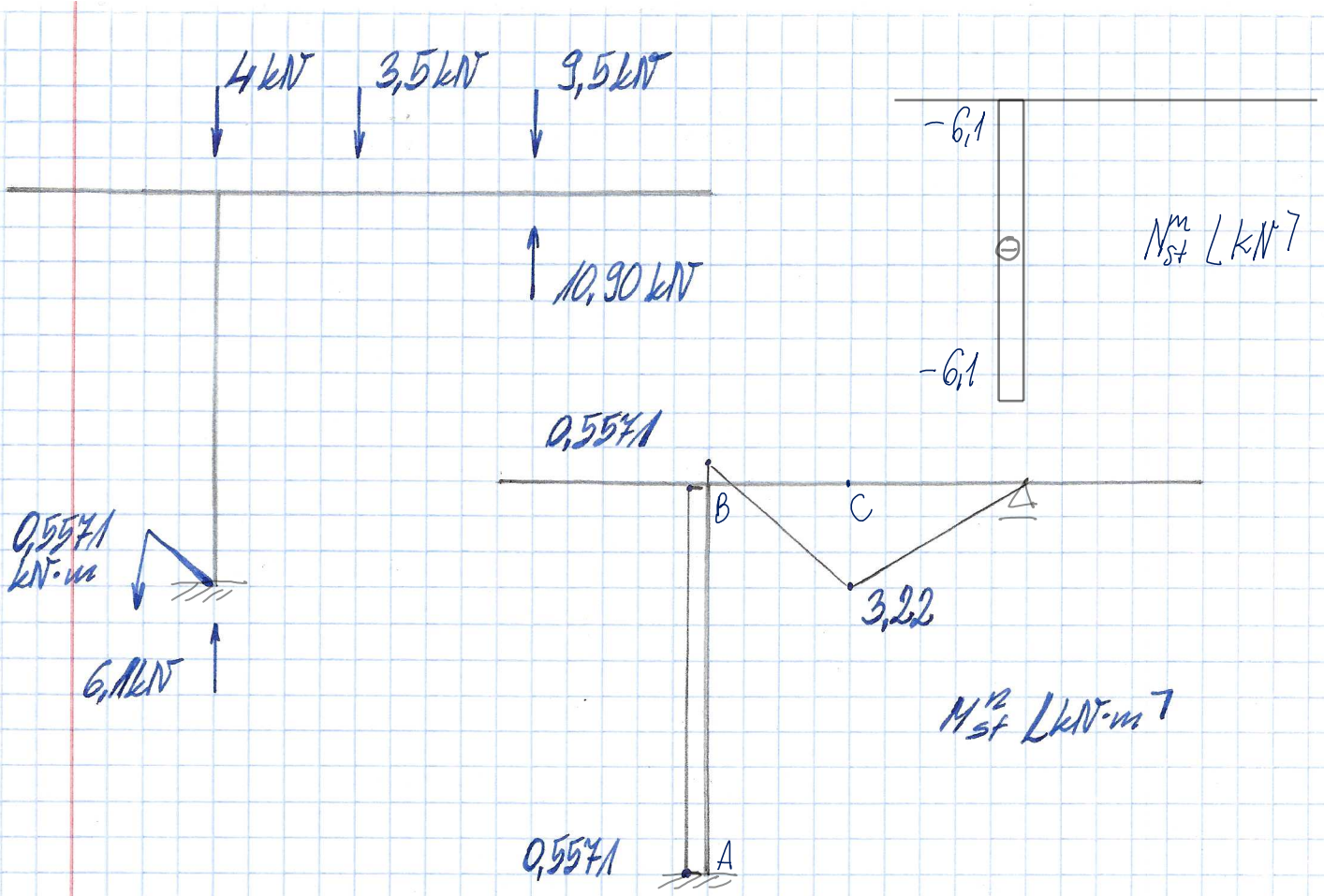
$$- \frac{1}{2} \cdot 1{,}8 \cdot 4{,}1 \cdot \left(\frac{2}{3} \cdot 45{,}25 + \frac{1}{3} \cdot 21{,}85\right) - \frac{1}{2} \cdot 1{,}8 \cdot 23 \left(\frac{2}{3} \cdot 21{,}85 + \frac{1}{3} \cdot 45{,}25\right)\Big] =$$

$$= \frac{-1240}{EJ}$$

$$\delta_{11} x_1 + \delta^{\circ}_{1st} = 0$$

$$\frac{113{,}7}{EJ} x_1 - \frac{1240}{EJ} = 0$$

$$x_1 = \frac{1240}{EJ} \cdot \frac{EJ}{113{,}7} = 10{,}90\ kN$$



$M_A = 1,2 \cdot 0,5571 + 5,0 \cdot 0,0318 = 0,8267\ kN \cdot m$

$M_B = 1,2 \cdot 0,5571 + 5,0 \cdot 0,0632 = 0,9848\ kN \cdot m$

$M_C = 1,2 \cdot 3,22 + 5,0 \cdot 0,1256 = 4,494\ kN \cdot m = M_{max}$

$N_C = 1,2 \cdot 0 + 5,0 \cdot 0,01343 = 0,06715\ kN$

$$\sigma_x = \frac{|M_{max}|}{W_y} + \frac{|N|}{A} = \frac{449,4}{34,2} + \frac{0,06715}{10,3} = 13,15\ \frac{kN}{cm^2} = 131,5\ MPa$$

Wniosek:

Naprężenia dopuszczalne nie zostały przekroczone.

					L1		L2		L3				
		Dł elementów			5,4000		1,8000		2,3000				
		M1		-4,1000		-4,1000		-2,3000		0,0000			
		M1p		5,4000		0,0000		0,0000		0,0000			
		M2p		1,8000		1,8000		0,0000		0,0000			
	11	113,7477		45,3870	45,3870	12,9150	6,0030	4,0557	0,0000				
	1p	-59,7780		-39,8520	-19,9260	0,0000	0,0000	0,0000	0,0000				
	2p	-45,5220		-19,9260	-19,9260	-4,4280	-1,2420	0,0000	0,0000				
	Vb=	x1p	0,5255		Va=	-0,5255		Ha=	-1,0000		Ma=	-3,245320197	
	Vb=	x2p	0,4002		Va=	0,5998		Ha=	0,0000		Ma=	-0,159173375	

				L1		L2		L3		
	Dł elementów			5,4000		1,8000		2,3000		
	M1n		3,2453		-2,1547		-1,2087		0,0000	
	M2n		0,1592		0,1592		-0,9205		0,0000	
11	21,0728		12,6644	2,0634	3,5669	1,6579	1,1201	0,0000		
12	2,3207		1,3947	-0,9260	0,3892	0,6098	0,8530	0,0000		
22	1,2220		0,0684	0,0684	-0,0288	0,4644	0,6496	0,0000		

Drgania własne

m1	400,0000	P1dyn	m1a1w^2cospt		
m2	350,0000	P2dyn	m2a2w^2cospt		
m3	950,0000	P3dyn	m3a3w^2cospt	α = m*w^2/EI	
m0	50,0000	P4dyn	0,0000		

	[cm^4]		[Gpa]		[N*m^2]		[Hz]	[rad/s]			
I	171,0000	E	210,0000	EI	359100,0000	p	24,5000	153,9380		delta	= b^2-4*a*c
											506278,0379
	a1	=	716,4741	αa1	16,2452	αa2					
	a2	=	78,9054	αa1	8,5542	αa2				pierw.delta	= 711,5321763
	det	=	1,0000	x^0	-725,0283	x^1	4846,9931	x^2			
										x1	= (-b-Pi.del)/(2*a)
	α1	=	0,001392214		0,0000						0,001392214
	α2	=	0,148190892		0,0000				Dane - analiza symulacji - szukaj wyniku		
									Rezonans	x2	= (-b+Pi.del)/(2*a
	w1	=	3,1621		p/w1	48,6822	NIE		TAK 1,25 < p/w < 0,75		0,148190892
	w2	=	32,6237		p/w2	4,7186	NIE		NIE p/w > 1,25 lub p/w < 0,75		

Postacie drgań własnych

I postać		0,0013922		a1	=	1,0000
0,0000	=	0,002514	a1	-0,022617	a2	
0,0000	=	-0,109853	a1	0,988091	a2	
	a2	=	0,111177	0,111177		
II postać		0,1482		a1	=	1,0000
0,0000	=	-105,1749	a1	-2,4074	a2	
0,0000	=	-11,6931	a1	-0,2676	a2	
	a2	=	-43,6883	-43,6883		

Warunek ortogonalności drgań

I i II postać	0,0000

Drgania wymuszone

m1	400,0000	P1dyn	m1a1w^2cospt		
m2	350,0000	P2dyn	m2a2w^2cospt		
m3	950,0000	P3dyn	m3a3w^2cospt		
P (t)	3400,0000	P4dyn	P (t)		

	[cm^4]		[Gpa]		[N*m^2]		[Hz]	[rad/s]					
I	171,0000	E	210,0000	EI	359100,0000	p	24,5000	153,9380					
									M2				
	a1	=	2363,9975	αa1	53,6010	αa2			0,0219731				
	a2	=	260,3475	αa1	28,2243	αa2			0,0115702				
	0,0000	=	2362,9975	αa1	53,6010	αa2			0,0219731				
	0,0000	=	260,3475	αa1	27,2243	αa2			0,0115702				
		a1	=	0,00000043618									
		a2	=	-0,00042916777									

							Fc	g	= 10,0000	m/s^2
P1dyn	m1a1w^2cospt		17,5714	0,0176			4,0000			
P2dyn	m2a2w^2cospt	P(t)	-159,4841	-0,1595			3,5000			
P4dyn	0,0000						9,5000			

Vb=				Va=	-0,1595		Ha=	-0,0176	Ma=	0,192185518
Vb=				Va=	17,0000		Ha=	0,0000	Ma=	-45,25

				L1		L2		L3		
	Dł elementów			5,4000		1,8000		2,3000		
	M1		-4,1000		-4,1000		-2,3000		0,0000	
	Mod		-0,1922		-0,2871		0,0000		0,0000	
	Mos		45,2500		45,2500		21,8500		0,0000	
11	113,7477		45,3870	45,3870	12,9150	6,0030	4,0557	0,0000		
1d	6,2096		2,4776	2,8278	0,7062	0,1981	0,0000	0,0000		
1s	-1239,9298		-500,9175	-500,9175	-138,1905	-61,3755	-38,5288	0,0000		
Vb=	x1d	-0,0546		Va=	-0,1049		Ha=	-0,0176	Ma=	-0,031639336
Vb=	x1s	10,9007		Va=	6,0993		Ha=	0,0000	Ma=	-0,557106812

			L1		L2		L3	
Dł elementów			5,4000		1,8000		2,3000	
Mnd		0,0316		-0,0632		0,1256		0,0000
Mns		0,5571		0,5571		-3,2216		0,0000
wsp dyn	5	0,8267		0,9848		4,4937		0,0000
wsp stat	1,2							

Naprężenia maksymalne		Mmax	4,494	kN*m	Wy	34,2	cm3
		σmax	13,140	kN/cm2			
			131,396	Mpa			