Politechnika Poznańska
Wydział Inżynierii Lądowej i Transportu
Instytut Analizy Konstrukcji
Zakład Mechaniki Budowli

Nazwisko i imię studenta: **Matuszewski Bartosz**
Nr albumu: **156366**
Rok akademicki: **2023/24**
Grupa / Grupa projektowa: **C1 / P1**

MECHANIKA BUDOWLI / SEM.4

ĆWICZENIE NR 1

METODA PRZEMIESZCZEŃ

Termin oddania: 25.04.2024

Data	Uwagi sprawdzającego	Podpis

Dla zadanej **RAMY:** **/ Nr schematu: 7**

1. Korzystając z metody przemieszczeń obliczyć i narysować wykresy sił przekrojowych (M, T, N) od zadanego obciążenia, wykonać kontrolę kinematyczną oraz statyczną.
2. Sprawdzić naprężenia w obu grupach przekrojów I_1 i I_2, porównać je z wartościami dopuszczalnymi naprężeń i sformułować wnioski (w przypadku niespełnienia warunku nośności, obliczeń nie trzeba powtarzać).

Dla zadanej **BELKI**: **/ Nr schematu: 1**

1. Korzystając z metody przemieszczeń obliczyć i narysować wykresy sił przekrojowych (M, T, N) od zadanego obciążenia.
2. Korzystając z metody przemieszczeń w ujęciu macierzowym obliczyć i narysować wykresy sił przekrojowych (M, T, N) od zadanego obciążenia.

W obliczeniach przyjąć: E = 210 GPa, f_y = 235 MPa.

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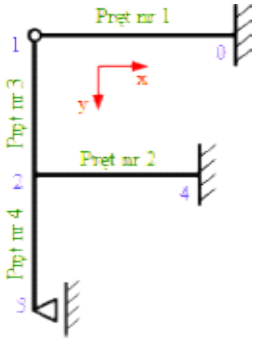
DANE DLA RAMY Nr schematu: 7

Wymiary i obciążenie przęsłowe:

Pręt	L [m]	Przekrój	q [kN/m]	P [kN]	M [kNm]
1	4	I 450 HEA	11	0	0
2	3,9	I 450 HEA	0	0	0
3	6	I 450 HEB	0	0	42
4	5	I 450 HEB	0	18	0

Obciążenie węzłowe:

siła skupiona pionowa w węźle 3	-10 kN
moment skupiony w węźle 2	15 kNm



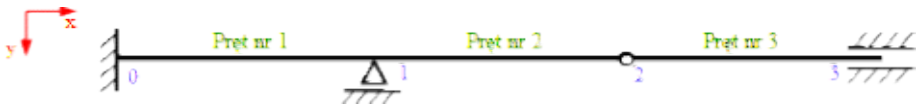
DANE DLA BELKI Nr schematu: 1

Wymiary i obciążenie przęsłowe:

Pręt	L [m]	Przekrój	q [kN/m]	P [kN]
1	6	I 360 PE	9	0
2	5,5	I 220 HEB	0	44
3	5,5	I 220 HEB	8	0

Obciążenie węzłowe:

siła skupiona pionowa w węźle 2	15 kN
moment skupiony w węźle 1	-35 kNm



LEGENDA

Obciążenie przęsłowe:

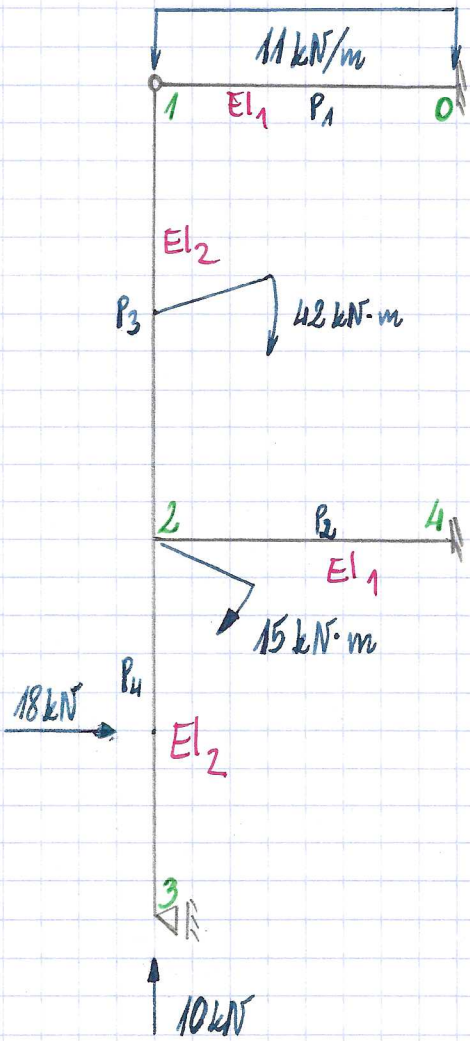
- obciążenie równomiernie rozłożone q działa na całej długości pręta, prostopadle do osi pręta;
- siła P prostopadła do osi pręta, przyłożona w połowie długości pręta;
- moment skupiony M przyłożony w połowie długości pręta.

Obciążenie przęsłowe i węzłowe – zwroty:

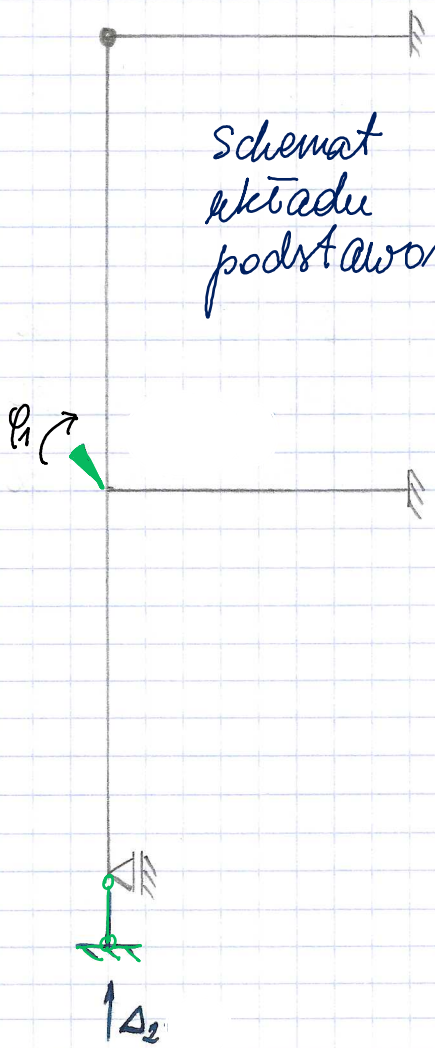
- siły skupione, obciążenie równomiernie rozłożone oraz liniowe osiadania podpór – wartość dodatnia oznacza zwrot zgodny z osią układu współrzędnych, który jest zaznaczony na schemacie, ujemna –
- momenty skupione oraz osiadania kątowe – wartość dodatnia oznacza zwrot w prawo, ujemna – w lewo.

Rama Schemat 4 $E = 210$ GPa $f_y = 235$ MPa

Pręt	L [m]	Przekrój		
1	4	I 450 HEA	$h_1 = 440$ mm	$b_1 = 300$ mm
2	3,9	I 450 HEA	$A_1 = 178$ cm^2	$I_{y_1} = 63720$ cm^4
3	6	I 450 HEB	$A_2 = 218$ cm^2	$I_{y_2} = 79890$ cm^4
4	5	I 450 HEB	$h_2 = 450$ mm	$b_2 = 300$ mm



SGN = 2 = 1 + 1



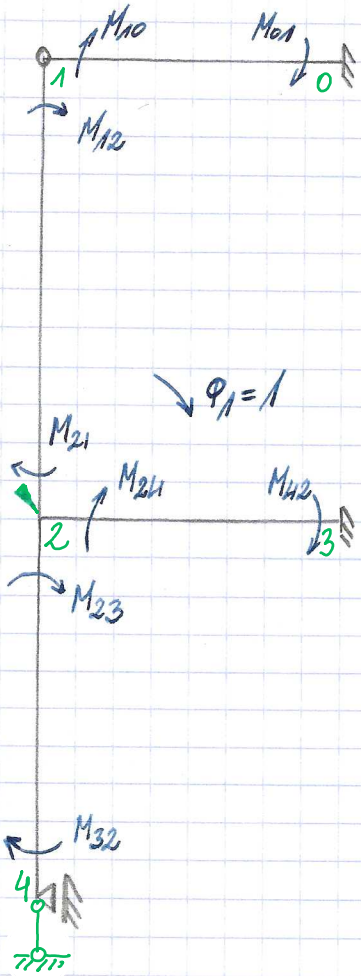
Schemat układu podstawowego

$$EJ_1 = 2{,}10 \cdot 63720 = 133812 \text{ kN} \cdot \text{m}^2$$

$$EJ_2 = 2{,}10 \cdot 79890 = 167769 \text{ kN} \cdot \text{m}^2$$

$$\frac{EJ_1}{EJ_2} = 0{,}7976$$

$$EJ_2 = EJ \qquad EJ_1 = 0{,}7976\, EJ$$



$M_{01} = 0 \quad M_{10} = 0 \quad M_{12} = 0$

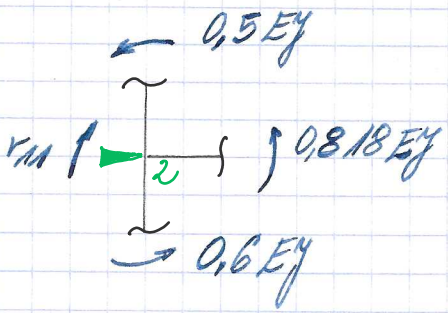
$M_{32} = 0$

$$M_{21} = \frac{3EJ_2}{6}(1-0) = \frac{EJ}{2}$$

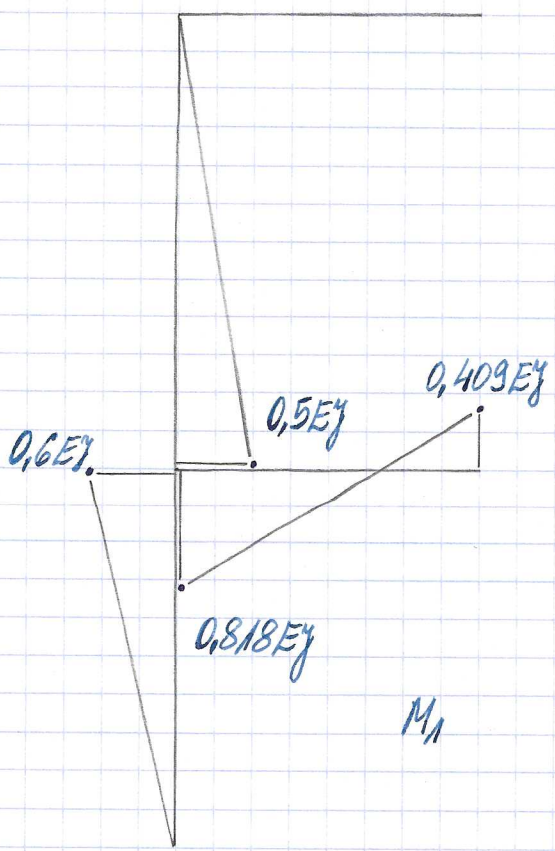
$$M_{24} = \frac{2EJ_1}{3{,}9}(2\cdot 1 + 1\cdot 0 - 3\cdot 0) = 0{,}8180\,EJ$$

$$M_{42} = \frac{2EJ_1}{3{,}9}(2\cdot 0 + 1\cdot 1 - 3\cdot 0) = 0{,}4090\,EJ$$

$$M_{23} = \frac{3EJ_2}{5}(1-0) = 0{,}6\,EJ$$



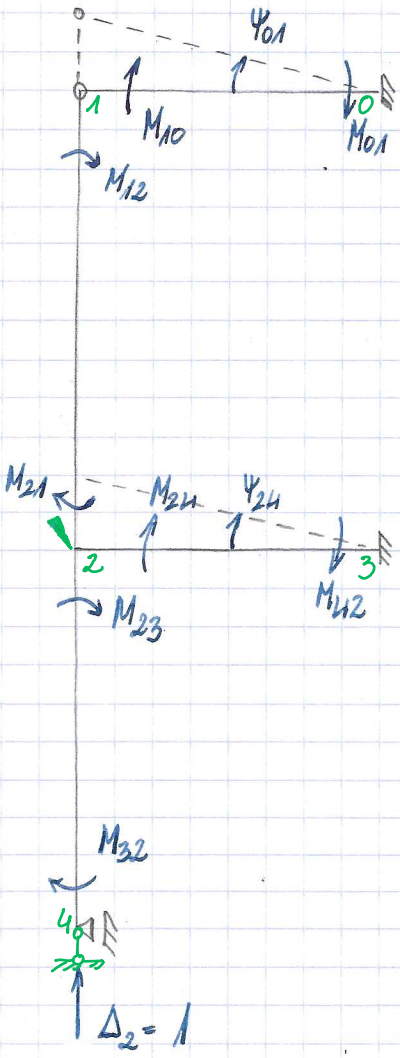
$r_{11} = 0{,}5EJ + 0{,}818EJ + 0{,}6EJ = 1{,}918EJ$



$$1\cdot r_{12} + \varphi_{24}(M_{24} + M_{42}) = 0$$

$$1\cdot r_{12} + \frac{1}{3{,}9}(0{,}818EJ + 0{,}409EJ) = 0$$

$$r_{12} = -\frac{1{,}227EJ}{3{,}9} = -0{,}3146\,EJ$$



$\Psi_{01} = \frac{1}{4}$ $\Psi_{24} = \frac{1}{3,9}$

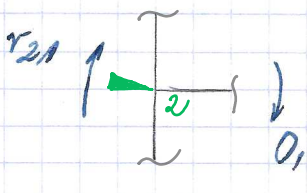
$M_{10} = 0$ $M_{12} = 0$ $M_{21} = 0$

$M_{23} = 0$ $M_{32} = 0$

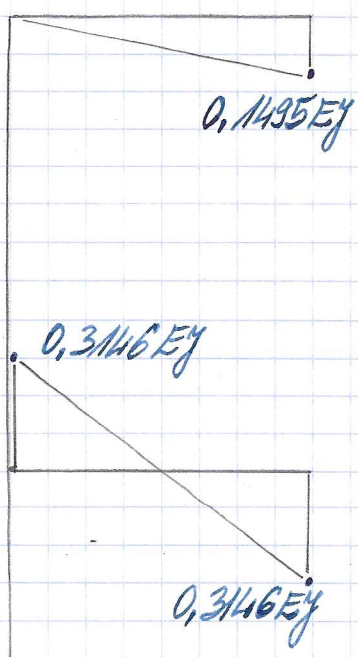
$$M_{01} = \frac{3EJ_1}{4,0}\left(0 - \frac{1}{4}\right) = -0,1495\,EJ$$

$$M_{24} = \frac{2EJ_1}{3,9}\left(2\cdot 0 + 1\cdot 0 - 3\cdot\frac{1}{3,9}\right) = -0,3146\,EJ$$

$$M_{42} = \frac{2EJ_1}{3,9}\left(2\cdot 0 + 1\cdot 0 - 3\cdot\frac{1}{3,9}\right) = -0,3146\,EJ$$



$r_{21} = -0,3146\,EJ$



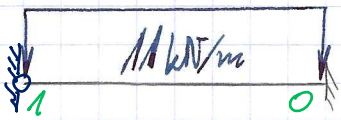
$$\bar{1}\cdot r_{22} + \Psi_{01}(M_{01} + M_{10}) + \Psi_{24}(M_{24} + M_{42}) = 0$$

$$\bar{1}\cdot r_{22} + \frac{1}{4}\cdot(-0,1495EJ + 0) + \frac{1}{3,9}(-0,3146\,EJ$$

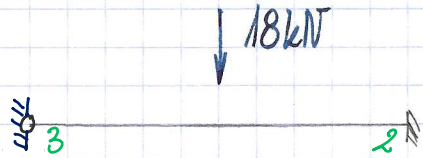
$$-0,3146\,EJ) = 0$$

$$\bar{1}\cdot r_{22} = \frac{1}{4}\cdot 0,1495EJ + \frac{2}{3,9}\cdot 0,3146EJ =$$

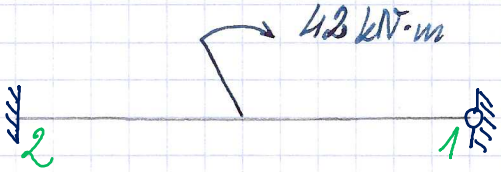
$$= 0,1987\,EJ$$

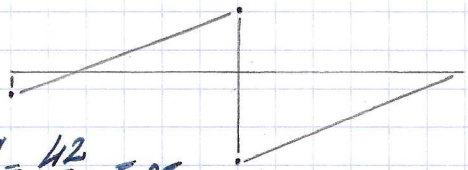


$$\frac{qL^2}{8} = \frac{11 \cdot 4^2}{8} = 22$$

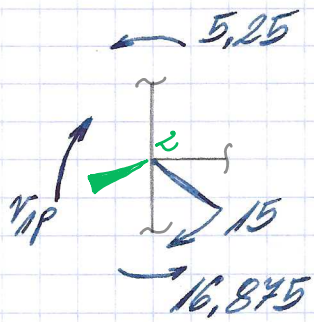


$$\frac{3}{16} P \cdot L = \frac{3}{16} \cdot 18 \cdot 5 = 16{,}875$$

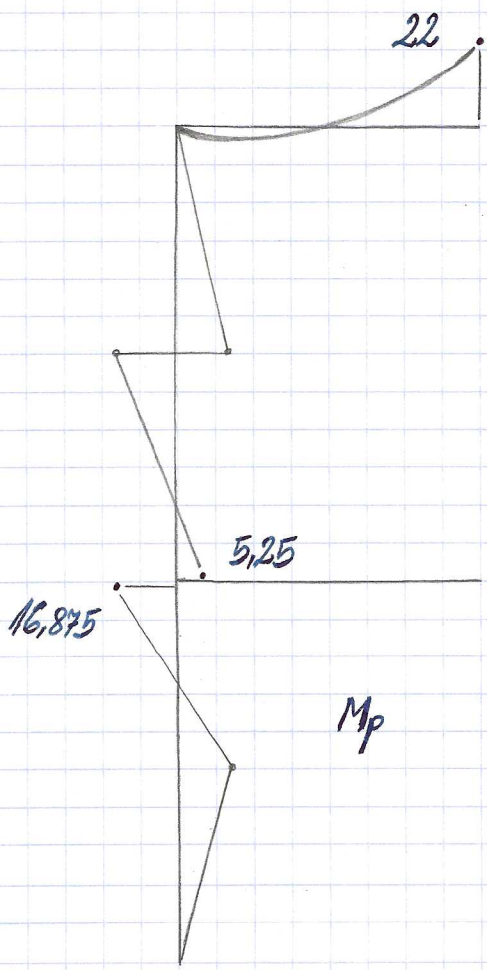




$$\frac{M}{8} = \frac{42}{8} = 5{,}25$$



$$r_{1p} = 5{,}25 + 16{,}875 - 15 = 7{,}125 \text{ kNm}$$



$$\bar{1} r_{2p} + 10 \cdot \bar{1} - 11 \cdot 4 \cdot \frac{\bar{1}}{2} + \psi_{01}(M_{01} + M_{10}) = 0$$

$$\bar{1} r_{2p} + 10 \cdot \bar{1} - 11 \cdot 4 \cdot \frac{\bar{1}}{2} + \frac{1}{4} \cdot 22 = 0$$

$$r_{2p} = -10 + 11 \cdot 4 \cdot \frac{1}{2} - \frac{1}{4} \cdot 22 = 6{,}5 \text{ kN}$$

Macierz sztywności

$$K = \begin{bmatrix} 1{,}918\,EJ & -0{,}3146\,EJ \\ -0{,}3146\,EJ & 0{,}1987\,EJ \end{bmatrix}$$

$$r_{1p} = 7{,}125 \text{ kNm}$$

$$r_{2p} = 6{,}5 \text{ kN}$$

$$W = \det K = 1{,}918\,EJ \cdot 0{,}1987\,EJ - (-0{,}3146\,EJ)^2 = 0{,}2822\,EJ^2$$

$$W_1 = \det \begin{bmatrix} -7{,}125 & -0{,}3146\,EJ \\ -6{,}5 & 0{,}1987\,EJ \end{bmatrix} = -7{,}125 \cdot 0{,}1987\,EJ - 6{,}5 \cdot 0{,}3146\,EJ =$$

$$= -3{,}461\,EJ$$

$$W_2 = \det \begin{bmatrix} 1{,}918\,EJ & -7{,}125 \\ -0{,}3146\,EJ & -6{,}5 \end{bmatrix} = -1{,}918\,EJ \cdot 6{,}5 - 7{,}125 \cdot 0{,}3146\,EJ =$$

$$= -14{,}71\,EJ$$

$$\varphi_1 = \frac{W_1}{W} = \frac{-3{,}461\,EJ}{0{,}2822\,EJ^2} = \frac{-12{,}27}{EJ}$$

$$\Delta_2 = \frac{W_2}{W} = \frac{-14{,}71\,EJ}{0{,}2822\,EJ^2} = \frac{-52{,}12}{EJ}$$

$$M_{01} = 0 \cdot \varphi_1 - 0{,}1495\,EJ \cdot \left(\frac{-52{,}12}{EJ}\right) + 22 = 29{,}80\ kN \cdot m$$

$$M_{10} = 0\ kN \cdot m$$

$$M_{12} = 0\ kN \cdot m$$

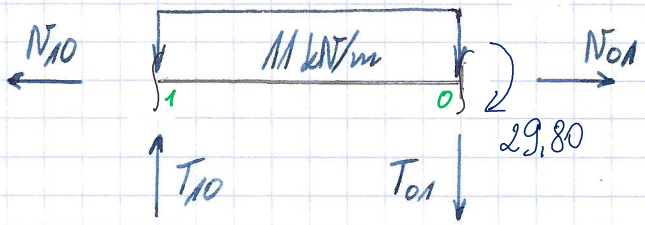
$$M_{21} = 0{,}5\,EJ \cdot \left(\frac{-12{,}27}{EJ}\right) + 0 \cdot \Delta_2 + 5{,}25 = -0{,}883\ kN \cdot m$$

$$M_{24} = 0{,}818\,EJ \cdot \left(\frac{-12{,}27}{EJ}\right) - 0{,}3146\,EJ \cdot \left(\frac{-52{,}12}{EJ}\right) + 0 = 6{,}367\ kN \cdot m$$

$$M_{42} = 0{,}409\,EJ \cdot \left(\frac{-12{,}27}{EJ}\right) - 0{,}3146\,EJ \cdot \left(\frac{-52{,}12}{EJ}\right) + 0 = 11{,}38\ kN \cdot m$$

$$M_{23} = 0{,}6\,EJ \cdot \left(\frac{-12{,}27}{EJ}\right) + 0 \cdot \Delta_2 + 16{,}875 = 9{,}516\ kN \cdot m$$

$$M_{32} = 0\ kN \cdot m$$



$\Sigma M_0 = 0$

$29,80 + 4 \cdot T_{10} - 11 \cdot 4 \cdot 2 = 0$

$$T_{10} = \frac{-29,80 + 11 \cdot 4 \cdot 2}{4} = 14,55\ kN$$

$\Sigma M_1 = 0$

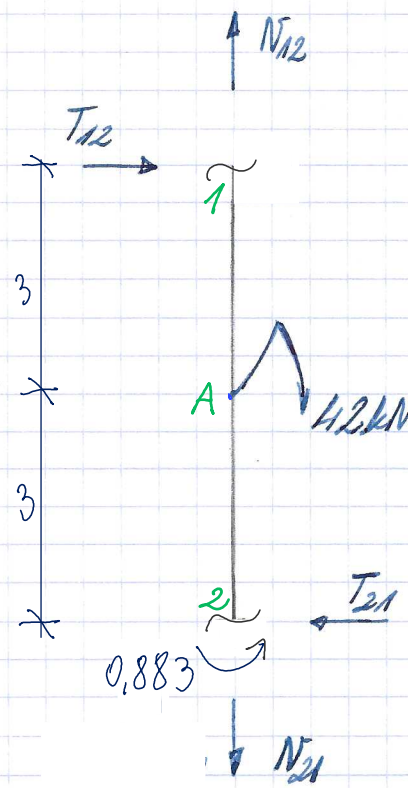
$11 \cdot 4 \cdot 2 + 4 \cdot T_{01} + 29,80 = 0$

$$T_{01} = \frac{-11 \cdot 4 \cdot 2 - 29,80}{4} = -29,45\ kN$$

$T_{10} - 11 \cdot x_0 = 0$

$$x_0 = \frac{T_{10}}{11} = \frac{14,55}{11} = 1,323\ m$$

$$M_{ekstr} = T_{10} \cdot x_0 - 11 \cdot x_0 \cdot \frac{x_0}{2} = 14,55 \cdot 1,323 - 11 \cdot 1,323 \cdot \frac{1,323}{2} = 9,624\ kN \cdot m$$



$\Sigma M_2 = 0$

$42 - 0,883 + 6T_{12} = 0$

$$T_{12} = \frac{-42 + 0,883}{6} = -6,853\ kN$$

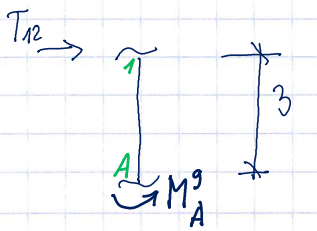
$\Sigma M_1 = 0$

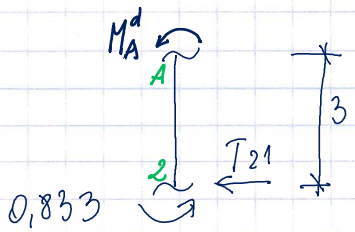
$42 - 0,883 + 6 \cdot T_{21} = 0$

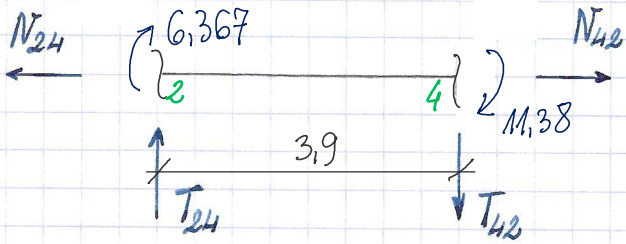
$$T_{21} = \frac{-42 + 0,883}{6} = -6,853\ kN$$

$M_A^g = 3T_{12} = 3 \cdot (-6,853) = -20,56\ kN \cdot m$

$M_A^d = -0,883 + 3 \cdot (-6,853) = -21,44\ kN \cdot m$







$\Sigma M2 = 0$

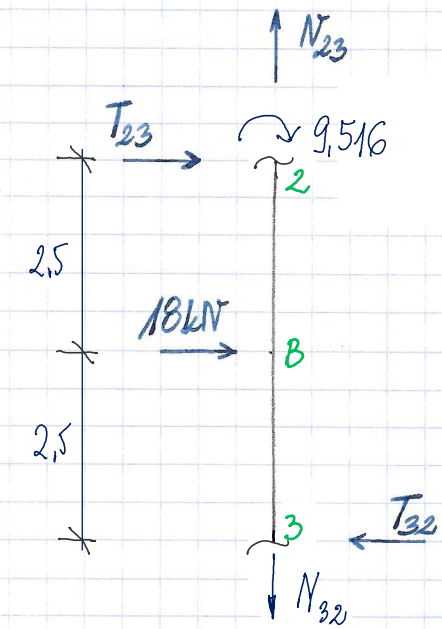
$6,367 + 11,38 + 3,9\,T_{42} = 0$

$$T_{42} = \frac{-6,367 - 11,38}{3,9} = -4,551\ kN$$

$\Sigma M4 = 0$

$6,367 + 11,38 + 3,9\,T_{24} = 0$

$$T_{24} = \frac{-6,367 - 11,38}{3,9} = -4,551\ kN$$



$\Sigma M3 = 0$

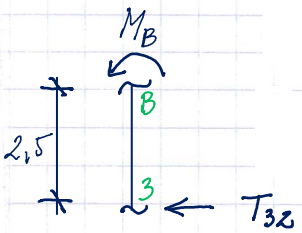
$5,0 \cdot T_{23} + 25 \cdot 18 + 9,516 = 0$

$$T_{23} = \frac{-25 \cdot 18 - 9,516}{5,0} = -10,90\ kN$$

$\Sigma M2 = 0$

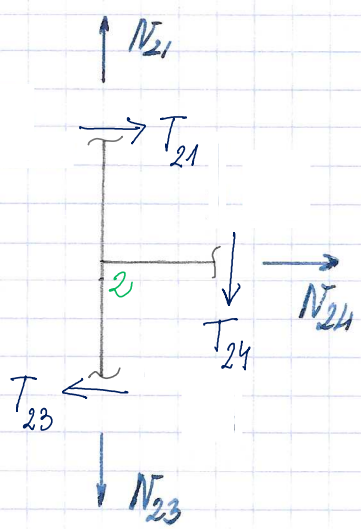
$9,516 - 18 \cdot 25 + 5,0 \cdot T_{32} = 0$

$$T_{32} = \frac{-9,516 + 18 \cdot 25}{5,0} = 7,097\ kN$$



$M_B = 25 \cdot T_{32} = 2,5 \cdot 7,097 = 17,74\ kNm$

$N_{23} = N_{32} = -10\ kN$



$\Sigma X = 0$

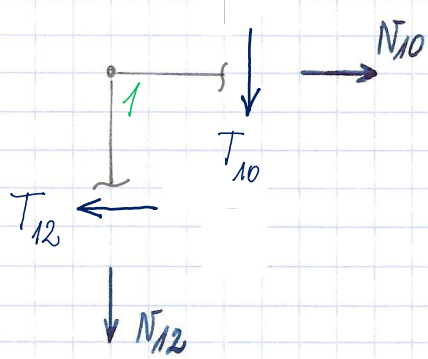
$T_{21} + N_{24} - T_{23} = 0$

$N_{24} = T_{23} - T_{21} = -10,90 + 6,853 = -4,047\ kN$

$\Sigma Y = 0$

$N_{21} - N_{23} - T_{24} = 0$

$N_{21} = N_{23} + T_{24} = -10 - 4,551 = -14,551\ kN$

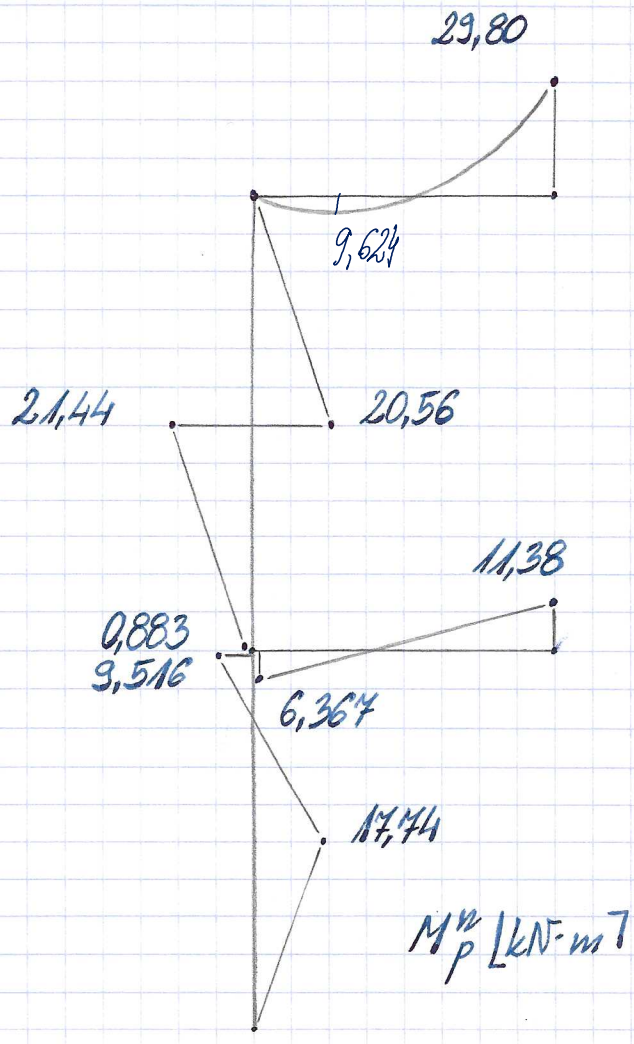


$$\Sigma X = 0$$

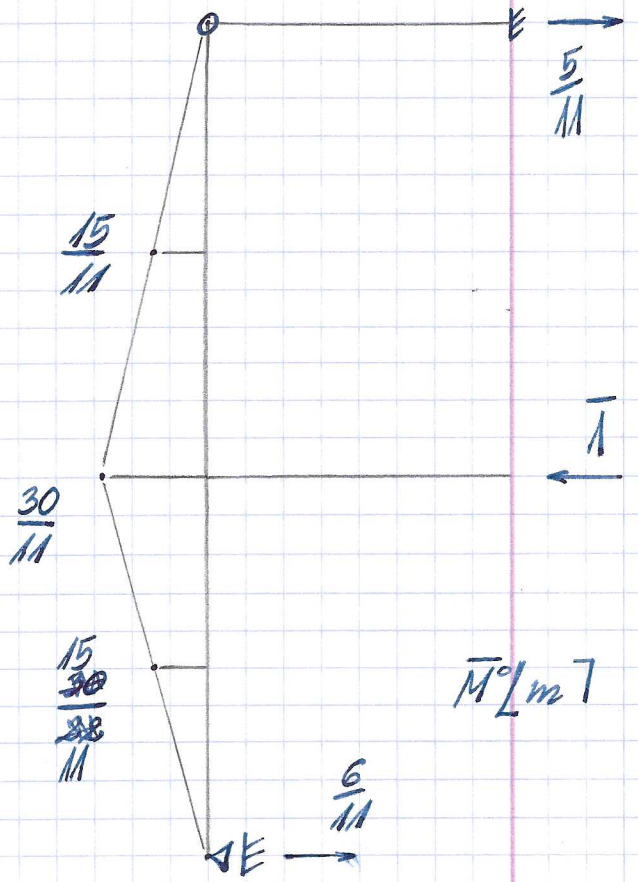
$$N_{10} = T_{12} = -6{,}853\ kN$$

$$\Sigma Y = 0$$

$$N_{12} = -T_{10} = -14{,}55\ kN$$



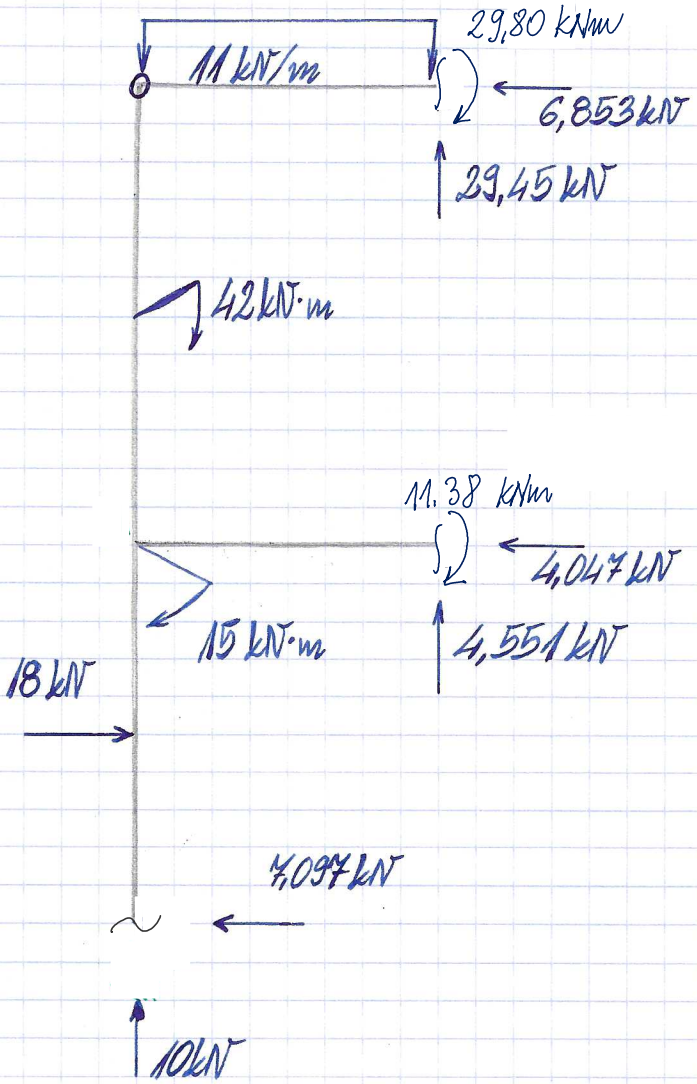
Kontrola kinematyczna:



$$\delta_{H\,"P"} = \Sigma \int \frac{M_P^n \bar{M}^0}{EJ}\,dx = \frac{1}{EJ_0}\left[-\frac{1}{2}\cdot 2{,}5\cdot 17{,}74\cdot \frac{1}{3}\cdot \frac{\frac{15}{11}}{\frac{30}{11}} - \frac{1}{2}\cdot 2{,}5\cdot 17{,}74 \cdot \left(\frac{2}{3}\cdot \frac{\frac{15}{11}}{\frac{30}{11}} + \frac{1}{3}\cdot \frac{30}{11}\right) + \frac{1}{2}\cdot 2{,}5\cdot 9{,}516\cdot \left(\frac{2}{3}\cdot \frac{30}{11} + \frac{1}{3}\cdot \frac{\frac{15}{11}}{\frac{30}{11}}\right) - \frac{1}{2}\cdot 3{,}0\cdot 20{,}56\cdot \frac{2}{3}\cdot \frac{15}{11} + \frac{1}{2}\cdot 3{,}0\cdot 21{,}44\cdot \left(\frac{2}{3}\cdot \frac{15}{11} + \frac{1}{3}\cdot \frac{30}{11}\right) + \frac{1}{2}\cdot 3{,}0\cdot 0{,}883\cdot \left(\frac{2}{3}\cdot \frac{30}{11} + \frac{1}{3}\cdot \frac{15}{11}\right)\right] = \frac{0{,}003}{EJ_2}$$

$$|\delta_{H\,"P"}| < \frac{1}{EJ_0} \Rightarrow OK$$

Sprawdzenie statyczne



$\Sigma X = 0$

$18 - 7{,}097 - 4{,}047 - 6{,}853 =$

$= 0{,}003 \cong 0$

$\Sigma Y = 0$

$10 + 4{,}551 + 29{,}45 - 11 \cdot 4{,}0 =$

$= 0{,}001 \cong 0$

$\Sigma M0 = 0$

$7{,}097 \cdot (6+5) + 10 \cdot 4 + 15$

$- 18 \cdot (6 + \frac{1}{2} \cdot 5) + 11{,}38$

$+ 4{,}047 \cdot 6 + 4{,}551 \cdot (4 - 3{,}9)$

$+ 42 + 29{,}80 - 11 \cdot 4 \cdot \frac{1}{2} \cdot 4 =$

$= -0{,}016 \cong 0$

$\Sigma M4 = 0$

$10 \cdot 3{,}9 + 7{,}097 \cdot 5 - 18 \cdot \frac{1}{2} \cdot 5 + 15 + 11{,}38 + 42 + 29{,}80 - 6{,}853 \cdot 6$

$- 29{,}45 \cdot 0{,}1 - 11 \cdot 4 \cdot (3{,}9 - \frac{1}{2} \cdot 4{,}0) = 0{,}002 \cong 0$

Sprawdzenie naprężeń:

Dla P1 P2

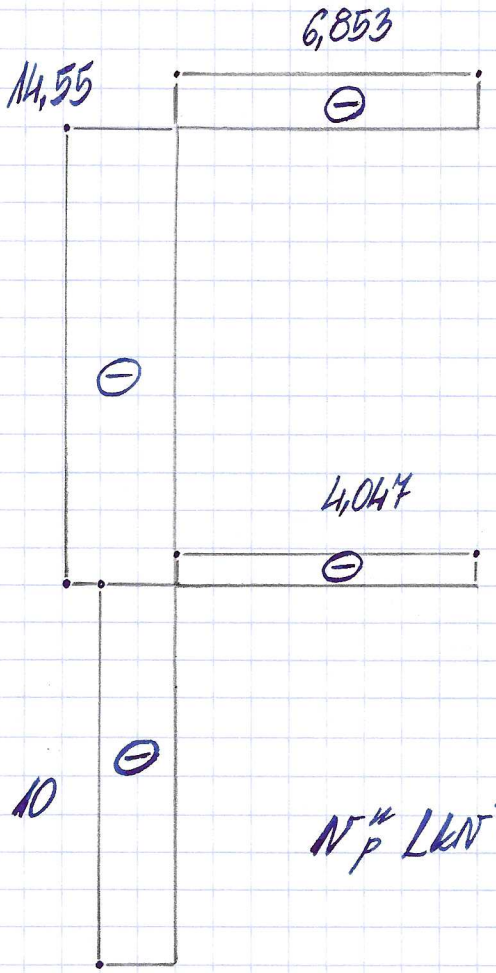
$$\sigma_x = \frac{M_{max}}{W_y} + \frac{N_{odp}}{A} = \frac{2380}{2896} + \frac{6{,}853}{178} = 1{,}067 \frac{kN}{cm^2} = 10{,}67\ MPa$$

Dla P3 P4

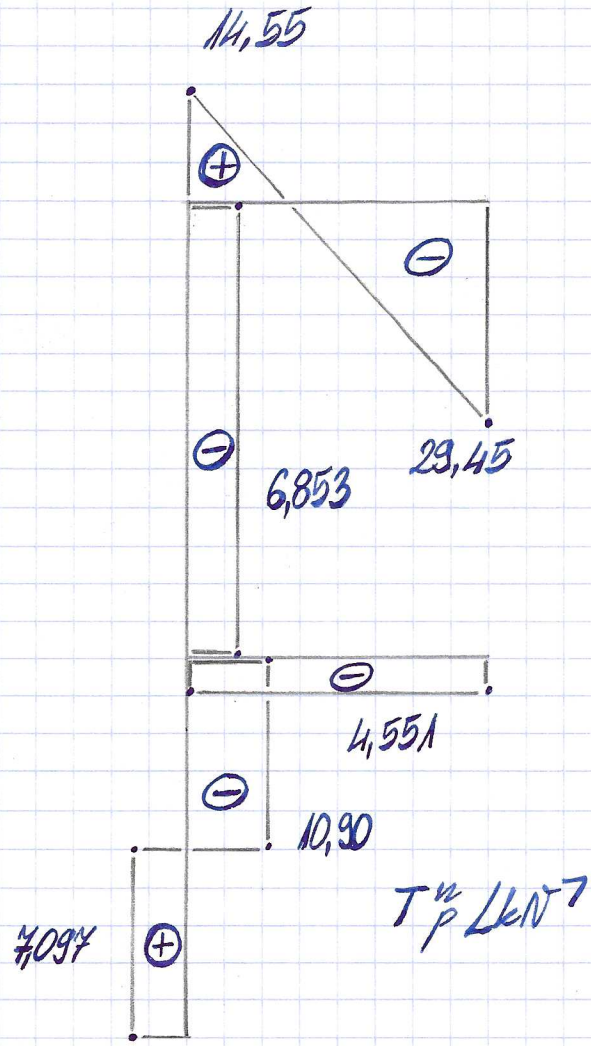
$$\sigma_x = \frac{M_{max}}{W_y} + \frac{N_{odp}}{A} = \frac{2144}{3551} + \frac{14{,}55}{218} = 0{,}671 \frac{kN}{cm^2} = 6{,}71\ MPa$$

Wniosek:

Naprężenia maksymalne nie zostały przekroczone



$N_p^{\prime\prime}$ [kN]



$T_p^{\prime\prime}$ [kN]

M nie usuwać
T nie usuwać

E = 210 Gpa
fy = 235 Mpa

Przekrój 1 — 450 HEA
h1 = 0,44 m
I1 = 63720,00 cm4
A1 = 178,00 cm2
EI1 = 133812,00 kN/m2

Przekrój 2 — 450 HEB
h2 = 0,45 m
I2 = 79890,00 cm4
A2 = 218,00 cm2
EI2 = 167769,00 kN/m2

EI0 = EI2 = 167769,0000 k1
EI1 = 0,7976 EI0 k1
EI2 = 1,0000 EI0 k2
k2

Moment w układzie wyznaczalnym od obciążeń P

	W		W	W		W	W		W		W		W		W
	1	L1	0	2	L2	4	2	L3	1	3	L4	2
Dl elementów		4,000			3,900			6,00			5,00	
EI	P	1	P	P	1	P	P	2	P	P	2	P
	2		3	3		3	3		2	2		3

Momenty na belkach elementarnych od obciążeń P

Moment węzłowy = 15
Siła węzłowa = 10

	P	M	Q	P	M	Q	P	M	Q	P	M	Q
	0	0	11	0	0	0	0	42	0	18	0	0
r 1p = 7,1250												
	M10		M01	M24		M42	M21		M12	M32		M23
r 2p = 6,50000	=		=	=		=	=		=	=		=
	0,0000		22,0000	0,0000		0,0000	5,2500		0,0000	0,0000		16,8750

Momenty na belkach elementarnych od ϕ1 = 1

	ϕ L	ϕ P	Ψ	ϕ L	ϕ P	Ψ	ϕ L	ϕ P	Ψ	ϕ L	ϕ P	Ψ
	0	0	0	1	0	0	1	0	0	0	1	0
		0,79760			0,79760			1,00000			1,00000	
r 11 = 1,9180												
	M10		M01	M24		M42	M21		M12	M32		M23
r 12 = -0,31463	=		=	=		=	=		=	=		=
	0,0000		0,0000	0,8180		0,4090	0,5000		0,0000	0,0000		0,6000

Momenty na belkach elementarnych od Δ2 = 1

	ϕ L	ϕ P	Ψ	ϕ L	ϕ P	Ψ	ϕ L	ϕ P	Ψ	ϕ L	ϕ P	Ψ
	0	0	0,25	0	0	0,256410256	0	0	0	0	0	0
		0,79760			0,79760			1,00000			1,00000	
r 21 = -0,3146												
	M10		M02	M24		M42	M21		M13	M32		M2
r 22 = 0,19874	=		=	=		=	=		=	=		=
	0,0000		-0,1495	-0,3146		-0,3146	0,0000		0,0000	0,0000		0,0000

Równanie metody przemieszczeń

$$K = \begin{vmatrix} r11 & r12 \\ r21 & r22 \end{vmatrix} = \begin{vmatrix} 1,9180 & -0,31463 \\ -0,3146 & 0,19874 \end{vmatrix} \times EI0 \qquad r1p = 7,1250 \qquad r2p = 6,50000$$

r 11 ϕ1 + r 12 Δ2 + r 1p = 0,0000
r 21 ϕ1 + r 22 Δ2 + r 2p = 0,0000

1,9180 ϕ1 -0,3146 Δ2 7,1250 = 0,0000
-0,3146 ϕ1 0,1987 Δ2 6,5000 = 0,0000

W = 0,2822 W1 = -3,4611 W2 = -14,7091

ϕ1 = -12,265 / EI Δ2 = -52,124 / EI

Moment w układzie niewyznaczalnym

	M10		M01	M24		M42	M21		M12	M32		M23
	=		=	=		=	=		=	=		=
	0,0000		22,0000	0,0000		0,0000	5,2500		0,0000	0,0000		16,8750
	+		+	+		+	+		+	+		+
	0,0000		0,0000	0,8180		0,4090	0,5000		0,0000	0,0000		0,6000
	x		x	x		x	x		x	x		x
	-12,265		-12,265	-12,265		-12,265	-12,265		-12,265	-12,265		-12,265
	+		+	+		+	+		+	+		+
	0,0000		-0,1495	-0,3146		-0,3146	0,0000		0,0000	0,0000		0,0000
	x		x	x		x	x		x	x		x
	-52,124		-52,124	-52,124		-52,124	-52,124		-52,124	-52,124		-52,124
	=		=	=		=	=		=	=		=
	0,000		29,795	6,367		11,383	-0,883		0,000	0,000		9,516

Siły tnące w układzie niewyznaczalnym

	T10		T01	T24		T42	T21		T12	T32		T23
	=		=	=		=	=		=	=		=
	14,551		-29,449	-4,551		-4,551	-6,853		-6,853	7,097		-10,903
0	M1	=	0,000	0 M1	=	0,000	0 M1	=	0,000	1 M1	=	-17,742
	M2	=	0,000	M2	=	0,000	M2	=	0,000	M2	=	17,742
0	M1	=	0,000	0 M1	=	0,000	1 M1	=	-20,559	0 M1	=	0,000
	M2	=	0,000	M2	=	0,000	M2	=	-21,441	M2	=	0,000
1	xo	=	1,323	0 xo	=	#DZIEL/0!	0 xo	=	#DZIEL/0!	0 xo	=	#DZIEL/0!
	Meks	=	9,624	Meks	=	#DZIEL/0!	Meks	=	#DZIEL/0!	Meks	=	#DZIEL/0!

Moment w układzie niewyznaczalnym

	W		W	W		W	W		W		W		W		W
	1	L1	0	2	L2	4	2	L3	1	3	L4	2
Dl elementów		4,000			3,900			6,00			5,00	
EI	P	1	P	P	1	P	P	2	P	P	2	P
	2		3	3		3	3		2	2		3

	P	M	Q	P	M	Q	P	M	Q	P	M	Q
	0	0	11	0	0	0	0	42	0	18	0	0
	Dl	4		Dl	3,9		Dl	3	Dl	3	Dl	2,5	Dl	2,5
	0,000	29,795	q 11	-6,367	11,383	0	0	0,883	21,441	-20,559	0,000	0,000	-17,742	-17,742	9,516

Moment w układzie wyznaczalnym do sprawdzenia kinematycznego

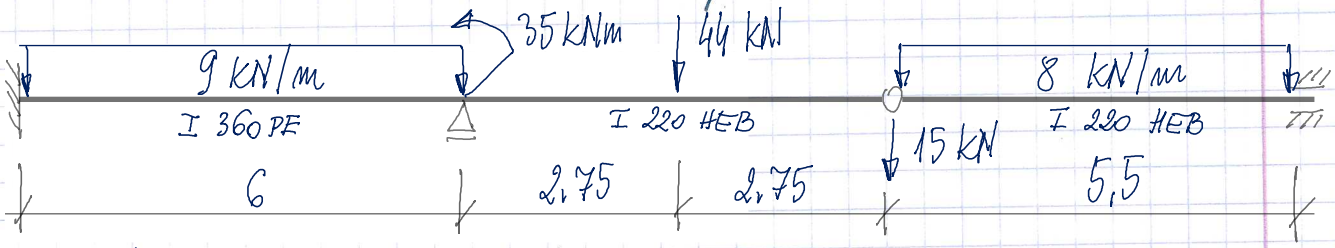
M	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	2,7273	1,3636	1,3636	0,0000	0,0000	1,3636	1,3636	2,7273

Sprawdzenie kinematyczne

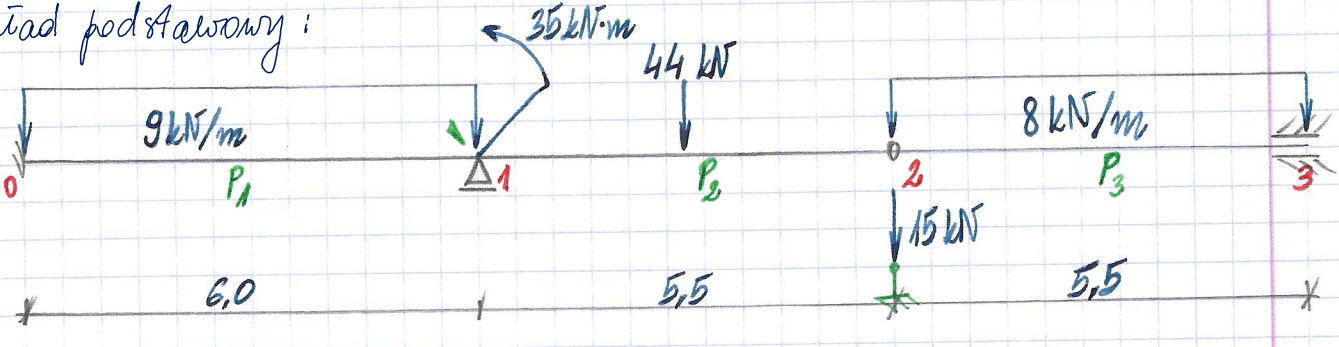
	0,0000	0,0000	0,0000		0,0000	0,0000	0,0000	0,0000	6,0172	116,9523	-56,0693	0,0000	0,0000	-40,3228	-80,6455	54,0680
	0,0000	0,0000	0,0000		0,0000	0,0000	0,0000	0,0000	6,0172	116,9523	-56,0693	0,0000	0,0000	-40,3228	-80,6455	54,0680
0,0000		0,0000		0,0000		0,0000		0,0000		0,0000		0,0000		0,0000		0,0000
/		0,0000		0,0000		0,0000		0,0000		122,9695		-56,0693		-40,3228		-26,5775
0,0000 /	0,0000 /	0,0000 /	/	0,000 /												
EI1	EI2	k1	k2	EI0												

Belka Schemat 1

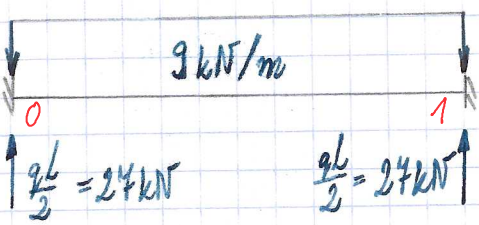
Pręt	Długość L [m]	Przekrój		J [cm⁴]
1	6	I 360 PE	q = 9 kN/m	16270
2	5,5	I 220 HEB	P = 44 kN	8090
3	5,5	I 220 HEB	q = 8 kN/m	8090

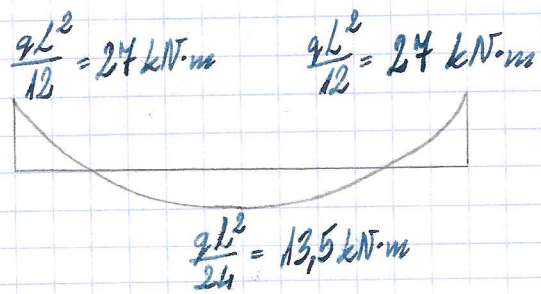


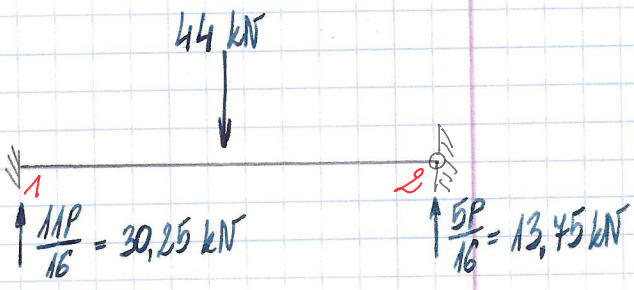
Układ podstawowy:

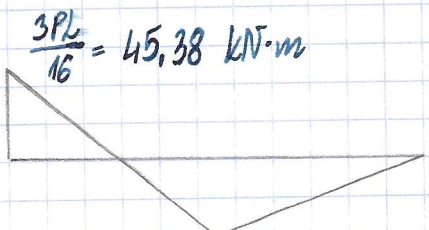


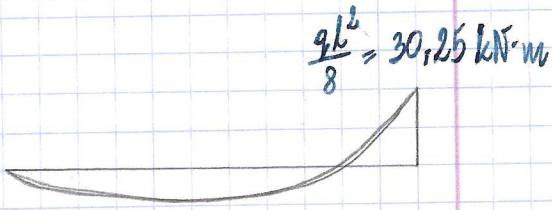
SGN = 1 + 1 = 2

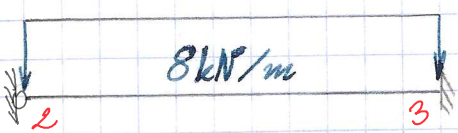




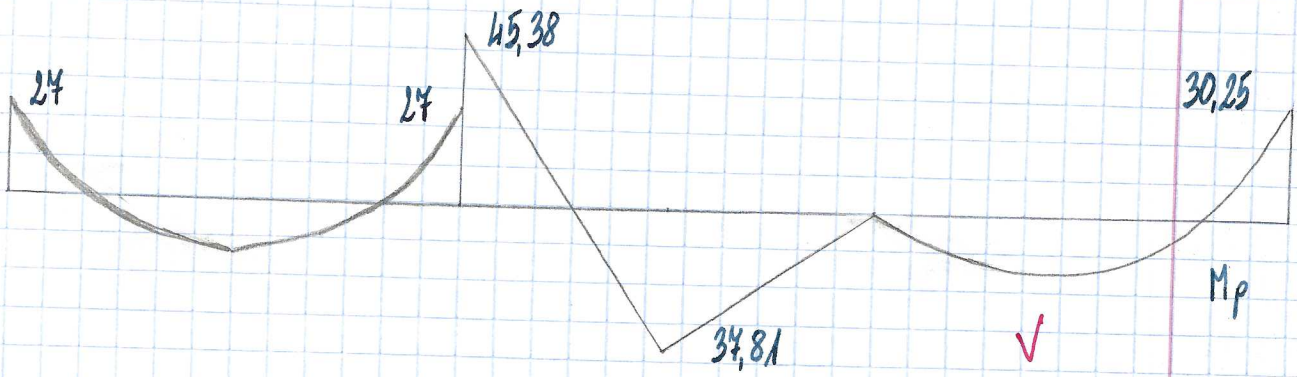








$\frac{3qL}{8}$ = 16,5 kN $\frac{5qL}{8}$ = 27,5 kN



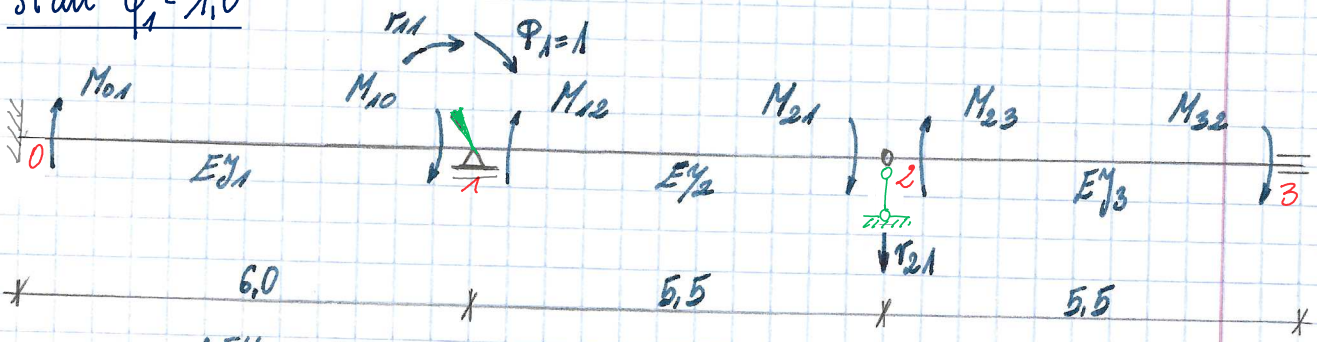
√

$EJ_1 = 2{,}10 \cdot 16240 = 34167\ kN\,m^2$ $\qquad EJ_1 = EJ$

$EJ_2 = EJ_3 = 210 \cdot 8090 = 16989\ kN\,m^2$

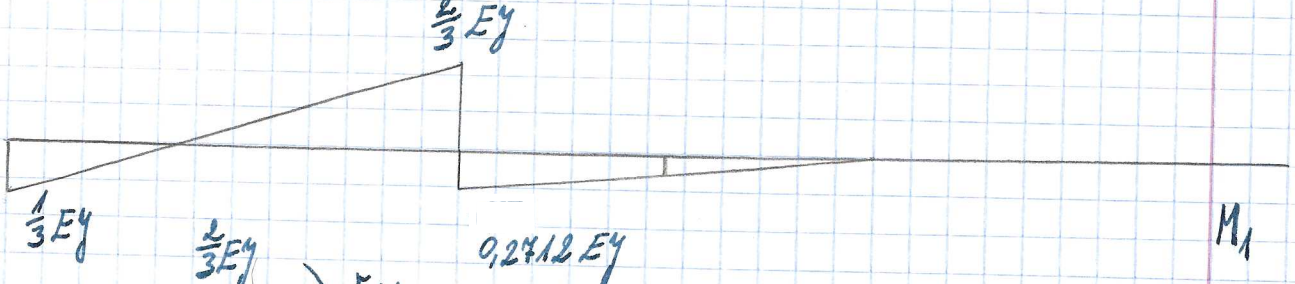
$\frac{EJ_2}{EJ_1} = \frac{16989}{34167} = 0{,}4972$ $\qquad EJ_3 = EJ_2 = 0{,}4972\,EJ$

Stan $\varphi_1 = 1{,}0$



$M_{01}^{(1)} = \frac{2EJ_1}{6}(1) = \frac{1}{3}EJ$ $\qquad M_{10}^{(1)} = \frac{2EJ_1}{6}(2 \cdot 1) = \frac{2}{3}EJ$

$M_{12}^{(1)} = \frac{3EJ_2}{5{,}5}(1) = 0{,}2712\,EJ$ $\qquad M_{21}^{(1)} = 0 = M_{23}^{(1)} = M_{32}^{(1)}$



$r_{11} = 0{,}2712\,EJ + 0{,}6667\,EJ = 0{,}9379\,EJ$

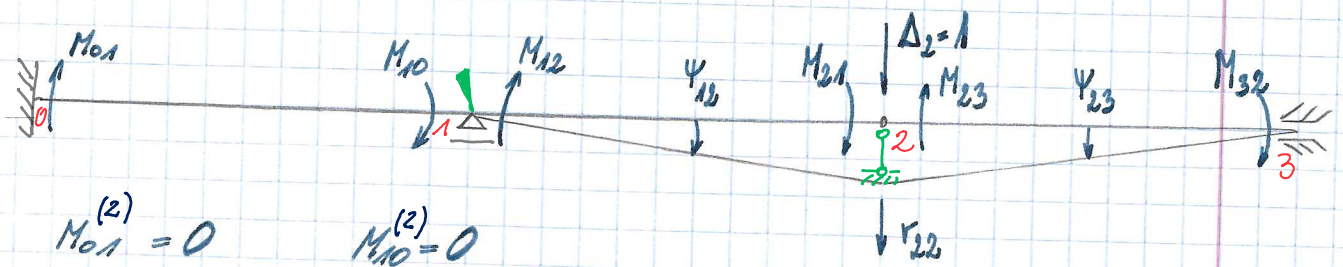
Stan $\Delta_2 = 1{,}0$

$$\Psi_{12} = \frac{1}{5{,}5} \qquad \Psi_{23} = -\frac{1}{5{,}5}$$

$$\sum_k P_k \bar{\delta}_k + \sum_j R_j \cdot \bar{\delta}_j + \sum_n M_{ik} \cdot \bar{\psi}_{ik} = 0$$

$$\bar{1} \cdot r_{21} + \bar{\Psi}_{12}\left(M_{12}^{(1)}\right) = 0 \qquad \bar{1} \cdot r_{21} + \frac{3EJ_2}{5{,}5} \cdot \frac{1}{5{,}5} = 0$$

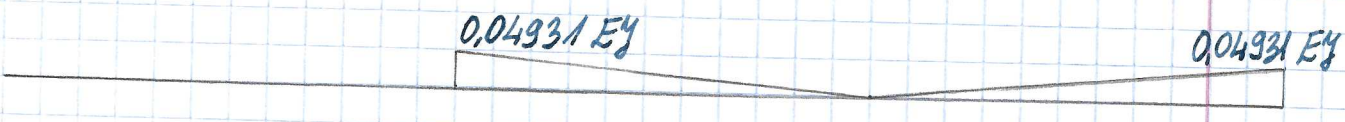
$$r_{21} = -\frac{1}{5{,}5} \cdot \left(\frac{3EJ_2}{5{,}5}\right) = -0{,}04931\ EJ$$



$$M_{01}^{(2)} = 0 \qquad M_{10}^{(2)} = 0$$

$$M_{12}^{(2)} = \frac{3EJ_2}{5{,}5}\left(-\frac{1}{5{,}5}\right) = -0{,}04931\ EJ \qquad M_{21}^{(2)} = 0$$

$$M_{23}^{(2)} = 0 \qquad M_{32}^{(2)} = \frac{3EJ_3}{5{,}5}\left(--\frac{1}{5{,}5}\right) = 0{,}04931\ EJ$$



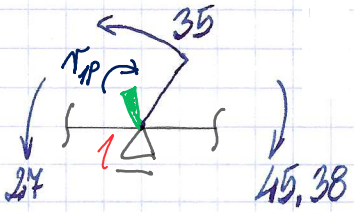
M_2

$$\bar{1} \cdot r_{22} + \bar{\Psi}_{12} \cdot M_{12}^{(2)} + \bar{\Psi}_{23} \cdot M_{32}^{(2)} = 0$$

$$r_{12} = -0{,}04931\ EJ$$

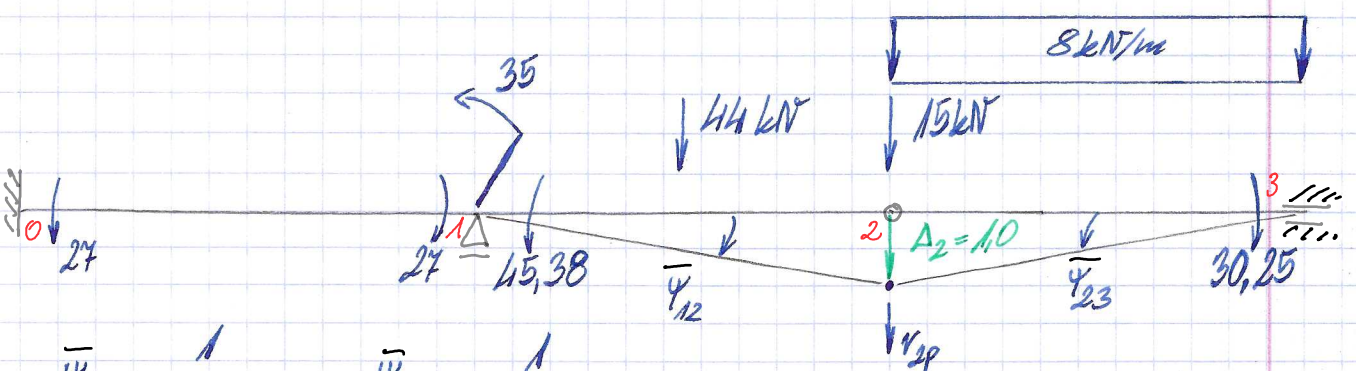
$$\bar{1} \cdot r_{22} + \frac{1}{5{,}5} \cdot (-0{,}04931\ EJ) - \frac{1}{5{,}5}(0{,}04931\ EJ) = 0$$

$$r_{22} = 0{,}01793\ EJ$$



$$r_{1p} + 45,38 - 27 - 35 = 0$$

$$r_{1p} = -45,38 + 27 + 35 = 16,62 \text{ kNm}$$



$$\bar{\psi}_{12} = \frac{1}{5,5} \qquad \bar{\psi}_{23} = -\frac{1}{5,5}$$

$$\bar{1} \cdot r_{2p} + 44 \cdot \frac{1}{2} \cdot 1 + 15 \cdot 1 + 8 \cdot 5,5 \cdot \frac{1}{2} \cdot 1 + \bar{\psi}_{12}(M_{12} + 0) + \bar{\psi}_{23}(0 + M_{32}) = 0$$

$$r_{2p} = -44 \cdot \frac{1}{2} - 15 \cdot 1 - 8 \cdot 5,5 \cdot \frac{1}{2} + \frac{1}{5,5} \cdot 45,38 + \frac{1}{5,5} \cdot 30,25 = -45,25 \text{ kN}$$

$r_{11} = 0,9379\,EJ$ $\quad r_{21} = -0,04931\,EJ$ $\quad r_{1p} = 16,62$ kNm

$r_{12} = -0,04931\,EJ$ $\quad r_{22} = 0,01793\,EJ$ $\quad r_{2p} = -45,25$ kN

Macierz sztywności

$$K = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} 0,9379\,EJ & -0,04931\,EJ \\ -0,04931\,EJ & 0,01793\,EJ \end{bmatrix}$$

$$W = \det K = [0,9379 \cdot 0,01793 - 0,04931^2]\,EJ^2 = 0,01439\,EJ^2$$

$$W_1 = \det \begin{bmatrix} -16,62 & -0,04931\,EJ \\ 45,25 & 0,01793\,EJ \end{bmatrix} = -16,62 \cdot 0,01793\,EJ - (-0,04931\,EJ) \cdot 45,25 =$$

$$= 1,933\,EJ$$

$$W_2 = \det \begin{bmatrix} 0,9379\,EJ & -16,62 \\ -0,04931\,EJ & 45,25 \end{bmatrix} = 0,9379\,EJ \cdot 45,25 - (-16,62) \cdot (-0,04931\,EJ) =$$

$$= 41,62\,EJ$$

$$\varphi_1 = \frac{W_1}{W} = \frac{1,933\,EJ}{0,01439\,EJ^2} = \frac{134,3}{EJ} \qquad \Delta_2 = \frac{W_2}{W} = \frac{41,62\,EJ}{0,01439\,EJ^2} = \frac{2892}{EJ}$$

$$M_{ik}^{n} = M_{ik}^{(1)} \cdot \varphi_1 + M_{ik}^{(2)} \cdot \Delta_2 + M_{ik}^{(P)}$$

$$M_{01} = \frac{1}{3} EJ \cdot \frac{134,3}{EJ} - 27 = 17,77 \text{ kN·m}$$

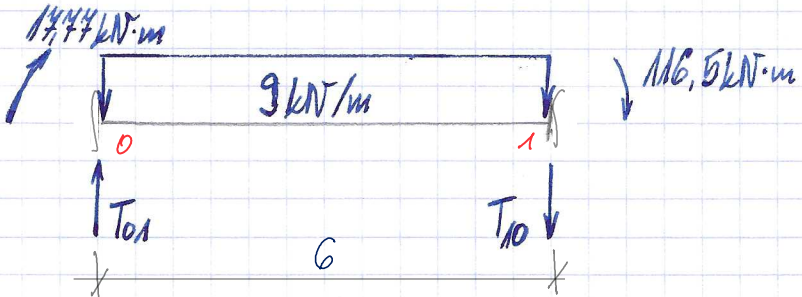
$$M_{10} = \frac{2}{3} EJ \cdot \frac{134,3}{EJ} + 27 = 116,5 \text{ kN·m}$$

$$M_{12} = 0,2712\, EJ \cdot \frac{134,3}{EJ} - 0,04931\, EJ \cdot \frac{2892}{EJ} - 45,38 = -151,6 \text{ kN·m}$$

$$M_{21} = 0 + 0 + 0 = 0 \text{ kN·m}$$

$$M_{23} = 0 + 0 + 0 = 0 \text{ kN·m}$$

$$M_{32} = 0 + 0,04931\, EJ \cdot \frac{2892}{EJ} + 30,25 = 172,9 \text{ kN·m}$$



$\Sigma M1 = 0$

$$6 \cdot T_{01} - 9 \cdot 6 \cdot 3 + 17,77 + 116,5 = 0$$

$$T_{01} = \frac{9 \cdot 6 \cdot 3 - 17,77 - 116,5}{6,0} = 4,621 \text{ kN}$$

$\Sigma M0 = 0$

$$9 \cdot 6 \cdot 3 + 17,77 + 116,5 + 6T_{10} = 0$$

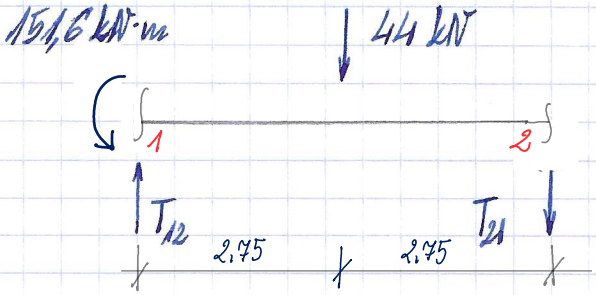
$$T_{10} = \frac{-9 \cdot 6 \cdot 3 - 17,77 - 116,5}{6,0} = -49,38 \text{ kN}$$

$$T_{01} - 9 \cdot x_0 = 0$$

$$x_0 = \frac{T_{01}}{9} = \frac{4,621}{9} = 0,5134 \text{ m}$$

$$M_{eks} = T_{01} \cdot x_0 + 17,77 - 9 \cdot x_0 \cdot \frac{x_0}{2} =$$

$$= 4,621 \cdot 0,5134 + 17,77 - 9 \cdot 0,5134 \cdot \frac{0,5134}{2} = 18,96 \text{ kN·m}$$



$\Sigma M1 = 0$

$44 \cdot \frac{5,5}{2} + 5,5 \cdot T_{21} - 151,6 = 0$

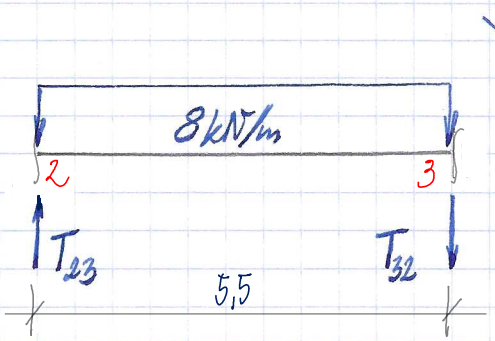
$T_{21} = \frac{-44 \cdot \frac{5,5}{2} + 151,6}{5,5} = 5,564\ kN$

$\Sigma M2 = 0$

$-151,6 - 44 \cdot \frac{5,5}{2} + 5,5\, T_{12} = 0$

$T_{12} = \frac{151,6 + 44 \cdot \frac{5,5}{2}}{5,5} = 49,56\ kN$

$T_{21} \cdot \frac{5,5}{2} = 5,564 \cdot \frac{5,5}{2} = 15,30\ kN \cdot m$



$\Sigma M2 = 0$

$8 \cdot 5,5 \cdot \frac{5,5}{2} + 172,9 + 5,5 \cdot T_{32} = 0$

$T_{32} = \frac{-8 \cdot 5,5 \cdot \frac{5,5}{2} - 172,9}{5,5} = -53,44\ kN$

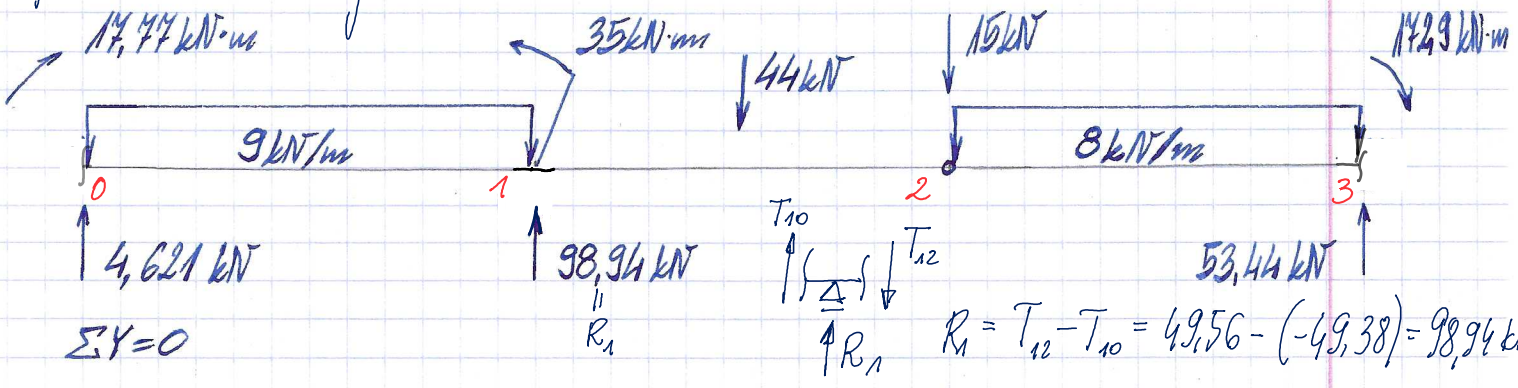
$\Sigma M3 = 0$

$5,5 \cdot T_{23} - 8 \cdot 5,5 \cdot \frac{5,5}{2} + 172,9 = 0$

$T_{23} = \frac{8 \cdot 5,5 \cdot \frac{5,5}{2} - 172,9}{5,5} = -9,436\ kN$

Brak zmiany znaku funkcji, brak momentu ekstremalnego w x_0.

Sprawdzenie statyczne



$R_1 = T_{12} - T_{10} = 49{,}56 - (-49{,}38) = 98{,}94$ kN

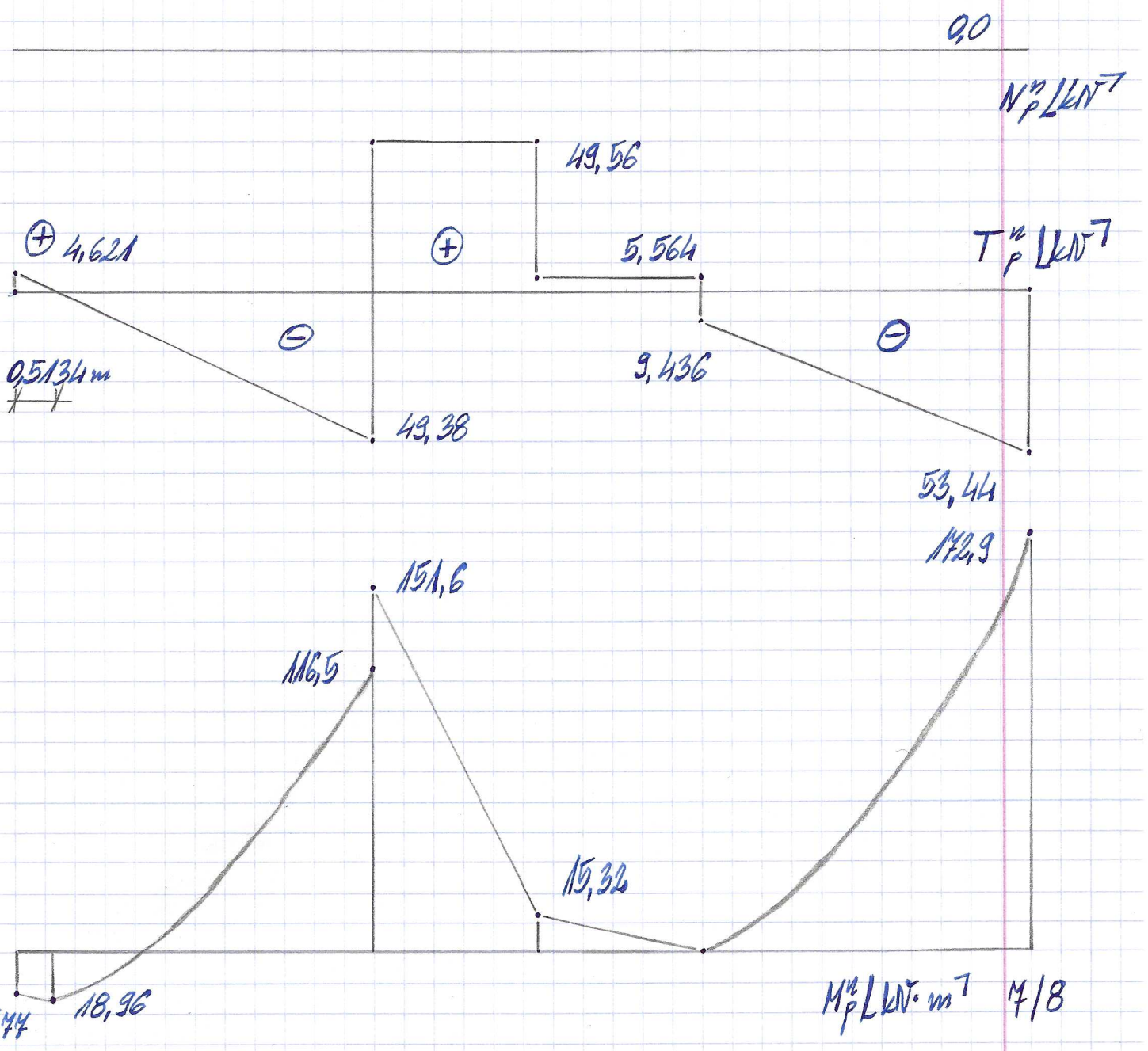
$\Sigma Y = 0$

$4{,}621 + 98{,}94 + 53{,}44 - 9 \cdot 6{,}0 - 44 - 15 - 8 \cdot 5{,}5 = 0{,}001 \cong 0$

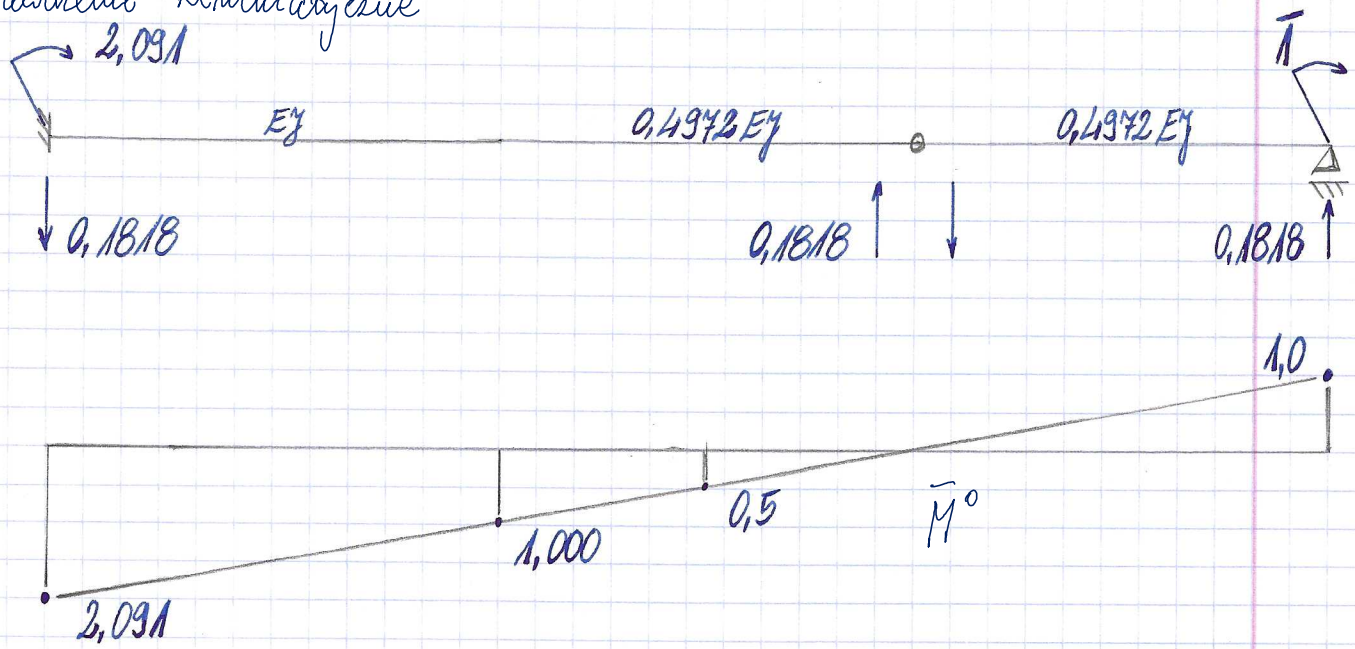
$\Sigma M0 = 0$

$17{,}77 + 9 \cdot 6 \cdot 3 - 35 - 98{,}94 \cdot 6{,}0 + 44 \cdot \left(6 + \frac{5{,}5}{2}\right) + 15 \cdot (6 + 5{,}5)$

$+ 8 \cdot 5{,}5 \cdot \left(6 + 5{,}5 + \frac{5{,}5}{2}\right) - 53{,}44 \cdot (6 + 5{,}5 + 5{,}5) + 172{,}9 = 0{,}05 \cong 0$



Sprawdzenie kinematyczne



$$\varphi = \sum \int \frac{M_p^n \bar{M}^0}{EJ} dx = \frac{1}{EJ}\left[\frac{1}{2}\cdot 6{,}0\cdot 17{,}77\cdot\left(\frac{2}{3}\cdot 2{,}091+\frac{1}{3}\cdot 1{,}0\right)-\frac{1}{2}\cdot 6{,}0\cdot 116{,}5\cdot\right.$$

$$\left.\left(\frac{2}{3}\cdot 1+\frac{1}{3}\cdot 2{,}091\right)+\frac{2}{3}\cdot\frac{9{,}0\cdot 6{,}0^2}{8}\cdot 6{,}0\cdot\left(\frac{1}{2}\cdot 2{,}091+\frac{1}{2}\cdot 1{,}0\right)\right]+$$

$$\frac{1}{0{,}4972\,EJ}\left[-\frac{1}{2}\cdot 2{,}75\cdot 151{,}6\cdot\left(\frac{2}{3}\cdot 1+\frac{1}{3}\cdot 0{,}5\right)-\frac{1}{2}\cdot 2{,}75\cdot 15{,}32\cdot\left(\frac{2}{3}\cdot 0{,}5+\frac{1}{3}\cdot 1\right)\right.$$

$$\left.-\frac{1}{2}\cdot 2{,}75\cdot 15{,}32\cdot\frac{2}{3}\cdot 0{,}5+\frac{1}{2}\cdot 5{,}5\cdot 172{,}9\cdot\frac{2}{3}\cdot 1-\frac{2}{3}\cdot\frac{8\cdot 5{,}5^2}{8}\cdot 5{,}5\cdot\frac{1}{2}\cdot 1{,}0\right]$$

$$= \frac{-134{,}1}{EJ}+\frac{66{,}75}{0{,}4972\,EJ} = \frac{0{,}1}{EJ}$$

$$|\varphi| < \frac{1}{EJ} \Rightarrow \varphi \approx 0 \Rightarrow \text{OK}$$

M nie usuwać

T nie usuwać

	E	=	210	Gpa	Przekrój 1		360 PE			Przekrój 2		220 HEB			EI0	=	EI1	=	34167,0000
	fy	=	235	Mpa	h1	=	0,00	m		h2	=	0,00	m		EI1	=	1,0000	EI0	
					I1	=	16270,00	cm4		I2	=	8090,00	cm4		EI2	=	0,4972	EI0	
					A1	=	0,00	cm2		A2	=	0,00	cm2						
					EI1	=	34167,00	kN/m2		EI2	=	16989,00	kN/m2						

Moment w układzie wyznaczalnym od obciążeń P

						W		W		W			W		W			W		
						0	L1	1		1	L2	2		2	L3	3				
					Dl elementów		6,000				5,500				5,50					
					EI	P	1	P		P	2	P		P	2	P				
						3		3		3		2		2		3				

Momenty na belkach elementarnych od obciążeń P

	Moment węzłowy	=	-35																	
	Siła węzłowa	=	15			P	M	Q		P	M	Q		P	M	Q				
						0	0	9		44	0	0		0	0	8				
	r1p	=	16,6250																	
						M01		M10		M12		M21		M23		M32				
	r 2p	=	-45,25000			=		=		=		=		=		=				
						-27,0000		27,0000		-45,3750		0,0000		0,0000		30,2500				

Momenty na belkach elementarnych od ϕ1 = 1

						ϕ L	ϕ P	Ψ		ϕ L	ϕ P	Ψ		ϕ L	ϕ P	Ψ				
						0	1	0		1	0	0		0	0	0				
							1,00000				0,49723				0,49723					
	r 11	=	0,9379																	
						M01		M10		M12		M21		M23		M32				
	r 12	=	-0,04931			=		=		=		=		=		=				
						0,3333		0,6667		0,2712		0,0000		0,0000		0,0000				

Momenty na belkach elementarnych od Δ2 = 1

						ϕ L	ϕ P	Ψ		ϕ L	ϕ P	Ψ		ϕ L	ϕ P	Ψ				
						0	0	0		0	0	0,181818182		0	0	-0,181818182				
							1,00000				0,49723				0,49723					
	r 21	=	-0,0493																	
						M01		M11		M12		M22		M23		M3				
	r 22	=	0,01793			=		=		=		=		=		=				
						0,0000		0,0000		-0,0493		0,0000		0,0000		0,0493				

Równanie metody przemieszczeń

	K	=	r11	r12	=	0,9379		-0,04931	x	EI0		r1p	=	16,6250						
			r21	r22		-0,0493		0,01793				r2p	=	-45,25000						
r 11	ϕ1	+	r 12	Δ2	+	r 1p	=	0,0000												
r 21	ϕ1	+	r 22	Δ2	+	r 2p	=	0,0000												
0,9379	ϕ1		-0,0493	Δ2		16,6250	=	0,0000												
-0,0493	ϕ1		0,0179	Δ2		-45,2500	=	0,0000												
W	=	0,0144		W1	=	1,9333		W2	=	41,6195										
ϕ1	=	134,383	/ EI			Δ2	=	2893,003	/	EI										

Moment w układzie niewyznaczalnym

						M01		M10		M12		M21		M23		M32				
						=		=		=		=		=		=				
						-27,0000		27,0000		-45,3750		0,0000		0,0000		30,2500				
						+		+		+		+		+		+				
						0,3333		0,6667		0,2712		0,0000		0,0000		0,0000				
						x		x		x		x		x		x				
						134,383		134,383		134,383		134,383		134,383		134,383				
						+		+		+		+		+		+				
						0,0000		0,0000		-0,0493		0,0000		0,0000		0,0493				
						x		x		x		x		x		x				
						2893,003		2893,003		2893,003		2893,003		2893,003		2893,003				
						=		=		=		=		=		=				
						17,794		116,589		-151,589		0,000		0,000		172,911				

Siły tnące w układzie niewyznaczalnym

						T01		T10		T12		T21		T23		T32				
						=		=		=		=		=		=				
						4,603		-49,397		49,562		5,562		-9,438		-53,438				
					0	M1	=	0,000	1	M1	=	15,294	0	M1	=	0,000				
						M2	=	0,000		M2	=	-15,294		M2	=	0,000				
					0	M1	=	0,000	0	M1	=	0,000	0	M1	=	0,000				
						M2	=	0,000		M2	=	0,000		M2	=	0,000				
					1	xo	=	0,511	0	xo	=	#DZIEL/0!	1	xo	=	-1,180				
						Meks	=	18,971		Meks	=	#DZIEL/0!		Meks	=	5,568				

Moment w układzie niewyznaczalnym

						W		W		W			W		W			W		
						0	L1	1		1	L2	2		2	L3	3				
					Dl elementów		6,000				5,500				5,50					
					EI	P	1	P		P	2	P		P	2	P				
						3		3		3		2		2		3				
						P	M	Q		P	M	Q		P	M	Q				
						0	0	9		44	0	0		0	0	8				
					Dl	6,000			Dl	2,75		2,75	Dl	5,50						
					-17,794	116,589	q	9	151,589	15,294	15,294	0	0,000	172,911	q	8				

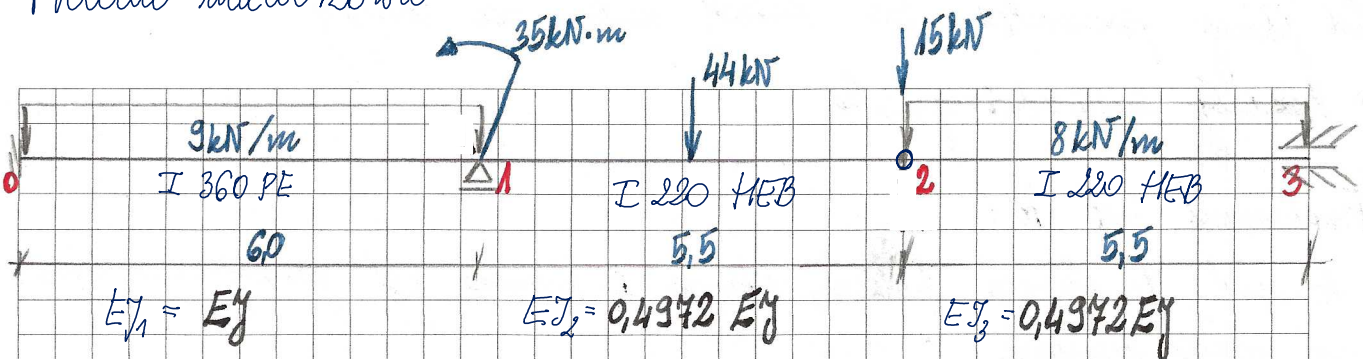
Moment w układzie wyznaczalnym do sprawdzenia kinematycznego

					M	-2,0909	-1,0000	-2,0909	-1,0000	-1,0000	-0,5000	-0,5000	0,0000	0,0000	1,0000	0,0000	1,0000			

Sprawdzenie kinematyczne

						92,2075	-476,9545	250,3636		-173,6956	-14,0199	-7,0099	0,0000	0,0000	317,0037	-55,4583				
						92,2075	-476,9545	250,3636		-173,6956	-14,0199	-7,0099	0,0000	0,0000	317,0037	-55,4583				
-134,3833	66,8200	0,0000			0,000		-384,7469		250,3636		0,0000		0,0000		0,0000		0,0000			
/	/	/	/		/		0,0000		0,0000		-187,7155		-7,0099		317,0037		-55,4583			
EI1	EI2	k1	k2		EI0															

Metoda macierzowa



$EJ_1 = EJ$ $\quad EJ_2 = 0{,}4972\,EJ$ $\quad EJ_3 = 0{,}4972\,EJ$

$EJ_1 = EJ$

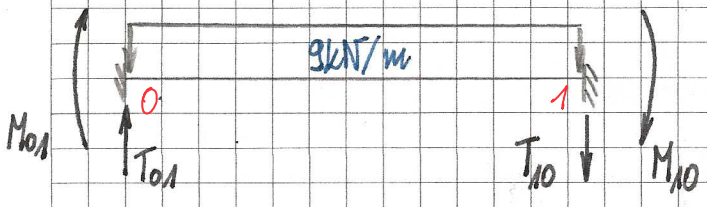
$$K_{e1} = \begin{bmatrix} \frac{12EJ_1}{L^3} & \frac{6EJ_1}{L^2} & -\frac{12EJ_1}{L^3} & \frac{6EJ_1}{L^2} \\ \frac{6EJ_1}{L^2} & \frac{4EJ_1}{L} & -\frac{6EJ_1}{L^2} & \frac{2EJ_1}{L} \\ -\frac{12EJ_1}{L^3} & -\frac{6EJ_1}{L^2} & \frac{12EJ_1}{L^3} & -\frac{6EJ_1}{L^2} \\ \frac{6EJ_1}{L^2} & \frac{2EJ_1}{L} & -\frac{6EJ_1}{L^2} & \frac{4EJ_1}{L} \end{bmatrix} = EJ \begin{bmatrix} 0{,}05556 & 0{,}16667 & -0{,}05556 & 0{,}16667 \\ 0{,}16667 & 0{,}66667 & -0{,}16667 & 0{,}33333 \\ -0{,}05556 & -0{,}16667 & 0{,}05556 & -0{,}16667 \\ 0{,}16667 & 0{,}33333 & -0{,}16667 & 0{,}66667 \end{bmatrix}$$

$$T_{01} = \frac{qL}{2} = 27\ kN$$

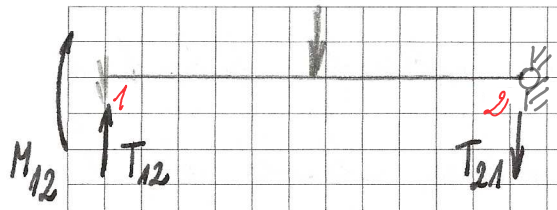
$$M_{01} = -\frac{qL^2}{12} = -27\ kNm$$

$$T_{10} = -\frac{qL}{2} = -27\ kN$$

$$M_{10} = -\frac{qL^2}{12} = 27\ kNm$$



$$R_e = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} -T_{01} \\ M_{01} \\ T_{10} \\ M_{10} \end{bmatrix} = \begin{bmatrix} -27 \\ -27 \\ -27 \\ 27 \end{bmatrix} \qquad q_{01} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$



$$T_{12} = \frac{11P}{16} = 30{,}25\ kN$$

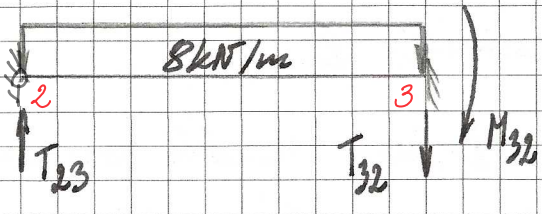
$$M_{12} = -\frac{3PL}{16} = -45{,}375\ kNm$$

$$T_{21} = -\frac{5P}{16} = -13{,}75\ kN$$

$$Re = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} -T_{12} \\ M_{12} \\ T_{21} \end{bmatrix} = \begin{bmatrix} -30{,}25 \\ -45{,}375 \\ -13{,}75 \end{bmatrix} \qquad q_{02} = \begin{bmatrix} q_3 \\ q_4 \\ q_5 \end{bmatrix}$$

$$EJ_2 = 0{,}4972\ EJ$$

$$Ke_2 = \begin{bmatrix} \frac{3EJ_2}{L^3} & \frac{3EJ_2}{L^2} & -\frac{3EJ_2}{L^3} \\ \frac{3EJ_2}{L^2} & \frac{3EJ_2}{L} & -\frac{3EJ_2}{L^2} \\ -\frac{3EJ_2}{L^3} & -\frac{3EJ_2}{L^2} & \frac{3EJ_2}{L^3} \end{bmatrix} = EJ \begin{bmatrix} 0{,}008965 & 0{,}049309 & -0{,}008965 \\ 0{,}049309 & 0{,}2712 & -0{,}049309 \\ -0{,}008965 & -0{,}049309 & 0{,}008965 \end{bmatrix}$$



$$T_{23} = \frac{3qL}{8} = 16{,}5\ kN$$

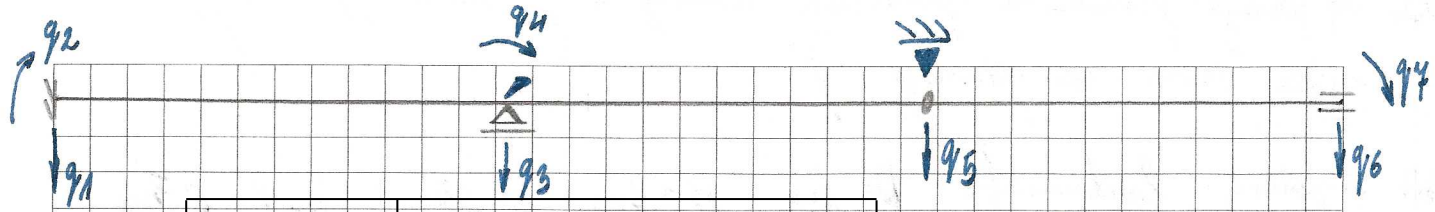
$$T_{32} = -\frac{5qL}{8} = -27{,}5\ kN$$

$$M_{32} = \frac{qL^2}{8} = 30{,}25\ kNm$$

$$Re = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} -T_{23} \\ T_{32} \\ M_{32} \end{bmatrix} = \begin{bmatrix} -16{,}5 \\ -27{,}5 \\ 30{,}25 \end{bmatrix} \qquad q_{03} = \begin{bmatrix} q_5 \\ q_6 \\ q_7 \end{bmatrix}$$

$$EJ_3 = 0{,}4972\ EJ$$

$$Ke_3 = \begin{bmatrix} \frac{3EJ_3}{L^3} & -\frac{3EJ_3}{L^3} & \frac{3EJ_3}{L^2} \\ -\frac{3EJ_3}{L^3} & \frac{3EJ_3}{L^3} & -\frac{3EJ_3}{L^2} \\ \frac{3EJ_3}{L^2} & -\frac{3EJ_3}{L^2} & \frac{3EJ_3}{L} \end{bmatrix} = EJ \begin{bmatrix} 0{,}008965 & -0{,}008965 & 0{,}049309 \\ -0{,}008965 & 0{,}008965 & -0{,}049309 \\ 0{,}049309 & -0{,}049309 & 0{,}2712 \end{bmatrix}$$



Tablica powiązań

Numer elementu	Numer przemieszczeń			
	1	2	3	4
1	1	2	3	4
2	3	4	5	–
3	5	6	7	–

Agregacja globalnego wektora reakcji węzłowych

$$R_o = \begin{bmatrix} R_{o11} \\ R_{o12} \\ R_{o13} + R_{o21} \\ R_{o14} + R_{o22} \\ R_{o23} + R_{o31} \\ R_{o32} \\ R_{o33} \end{bmatrix} = \begin{bmatrix} -27 \\ -27 \\ -27 - 30{,}25 \\ 27 - 45{,}375 \\ -13{,}75 - 16{,}5 \\ -27{,}5 \\ 30{,}25 \end{bmatrix} = \begin{bmatrix} -27 \\ -27 \\ -57{,}25 \\ -18{,}375 \\ -30{,}25 \\ -27{,}5 \\ 30{,}25 \end{bmatrix}$$

$$K = EJ \begin{bmatrix} 0{,}05556 & 0{,}16667 & -0{,}05556 & 0{,}16667 & 0 & 0 & 0 \\ 0{,}16667 & 0{,}66667 & -0{,}16667 & 0{,}33333 & 0 & 0 & 0 \\ -0{,}05556 & -0{,}16667 & 0{,}06453 & -0{,}11736 & -0{,}008965 & 0 & 0 \\ 0{,}16667 & 0{,}33333 & -0{,}11736 & 0{,}93787 & -0{,}049309 & 0 & 0 \\ 0 & 0 & -0{,}008965 & -0{,}049309 & 0{,}01793 & -0{,}008965 & 0{,}049309 \\ 0 & 0 & 0 & 0 & -0{,}008965 & 0{,}008965 & -0{,}049309 \\ 0 & 0 & 0 & 0 & 0{,}049309 & -0{,}049309 & 0{,}2712 \end{bmatrix}$$

Agregacja globalnego wektora sił węzłowych

$$P_{zs} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -35 \\ 15 \\ 0 \\ 0 \end{bmatrix}$$

$$P_{red} = P_{zs\,red} - R_{o\,red} = \begin{bmatrix} -35 \\ 15 \end{bmatrix} - \begin{bmatrix} -18{,}375 \\ -30{,}25 \end{bmatrix} = \begin{bmatrix} -16{,}625 \\ 45{,}25 \end{bmatrix}$$

$$K_{red}\, q_{red} = P_{red}$$

$$EJ \begin{bmatrix} 0{,}93787 & -0{,}049309 \\ -0{,}049309 & 0{,}01793 \end{bmatrix} \frac{1}{EJ} \begin{bmatrix} q_4 \\ q_5 \end{bmatrix} = \begin{bmatrix} -16{,}625 \\ 45{,}25 \end{bmatrix}$$

$$q_4 = \frac{134{,}394}{EJ}$$

$$q_5 = \frac{2893{,}305}{EJ}$$

$$q_{red} = \frac{1}{EJ} \begin{bmatrix} 134{,}394 \\ 2893{,}305 \end{bmatrix}$$

$$q_{01} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \frac{1}{EJ} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 134{,}394 \end{bmatrix}$$

$$R_1 = K_1 q_{01} + R_{01}$$

$$EJ \begin{bmatrix} 0{,}05556 & 0{,}16667 & -0{,}05556 & 0{,}16667 \\ 0{,}16667 & 0{,}66667 & -0{,}16667 & 0{,}33333 \\ -0{,}05556 & -0{,}16667 & 0{,}05556 & -0{,}16667 \\ 0{,}16667 & 0{,}33333 & -0{,}16667 & 0{,}66667 \end{bmatrix} \frac{1}{EJ} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 134{,}394 \end{bmatrix} + \begin{bmatrix} -27 \\ -27 \\ -27 \\ 27 \end{bmatrix} =$$

$$\begin{bmatrix} -4{,}601 \\ 17{,}738 \\ -49{,}399 \\ 116{,}596 \end{bmatrix} = \begin{bmatrix} -T_{01} \\ M_{01} \\ T_{10} \\ M_{10} \end{bmatrix}$$

$T_{01} = 4{,}601$ kN

$M_{01} = 17{,}738$ kN·m

$T_{10} = -49{,}399$

$M_{10} = 116{,}596$ kN·m

$$q_{02} = \begin{bmatrix} q_3 \\ q_4 \\ q_5 \end{bmatrix} = \frac{1}{EJ} \begin{bmatrix} 0 \\ 134,334 \\ 2893,305 \end{bmatrix} \qquad R_2 = K_2 q_{02} + R_{02}$$

$$EJ \begin{bmatrix} 0,008965 & 0,049303 & -0,008965 \\ 0,049303 & 0,2712 & -0,049303 \\ -0,008965 & -0,049303 & 0,008965 \end{bmatrix} \frac{1}{EJ} \begin{bmatrix} 0 \\ 134,334 \\ 2893,305 \end{bmatrix} + \begin{bmatrix} -30,25 \\ -45,375 \\ -13,75 \end{bmatrix} =$$

$$= \begin{bmatrix} -43,562 \\ -151,594 \\ 5,5611 \end{bmatrix} = \begin{bmatrix} -T_{12} \\ M_{12} \\ T_{21} \end{bmatrix}$$

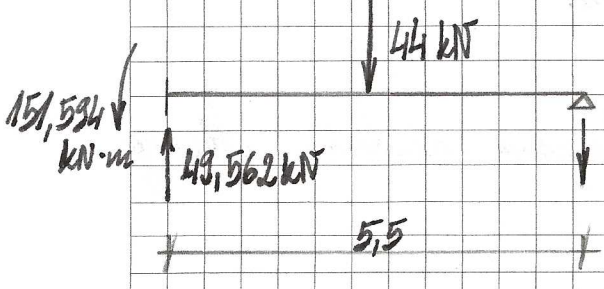
$$T_{12} = 43,562 \text{ kN}$$
$$M_{12} = -151,594 \text{ kN·m}$$
$$T_{21} = 5,5611 \text{ kN}$$

$$q_{03} = \begin{bmatrix} q_5 \\ q_6 \\ q_7 \end{bmatrix} = \frac{1}{EJ} \begin{bmatrix} 2893,305 \\ 0 \\ 0 \end{bmatrix} \qquad R_3 = K_3 q_{03} + R_{03}$$

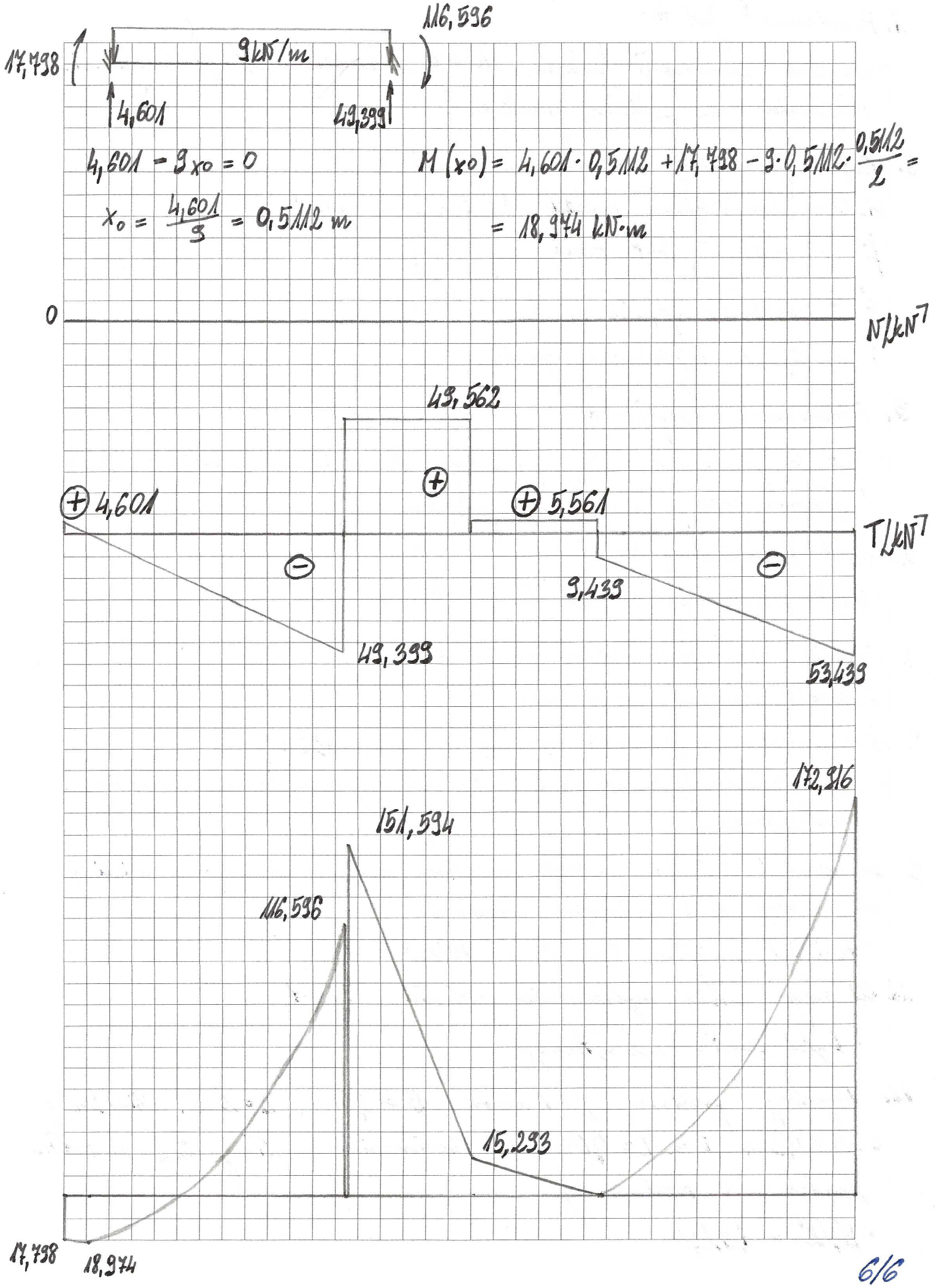
$$EJ \begin{bmatrix} 0,008965 & -0,008965 & 0,049303 \\ -0,008965 & 0,008965 & -0,049303 \\ 0,049303 & -0,049303 & 0,2712 \end{bmatrix} \frac{1}{EJ} \begin{bmatrix} 2893,305 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -16,5 \\ -27,5 \\ 30,25 \end{bmatrix} =$$

$$= \begin{bmatrix} 9,439 \\ -53,439 \\ 172,916 \end{bmatrix} = \begin{bmatrix} -T_{23} \\ T_{32} \\ M_{32} \end{bmatrix}$$

$$T_{23} = -9,439 \text{ kN}$$
$$T_{32} = -53,439 \text{ kN}$$
$$M_{32} = 172,916 \text{ kN·m}$$



$$M\left(\frac{5,5}{2}\right) = 5,561 \cdot \frac{5,5}{2} = 15,293 \text{ kN·m}$$



$4,601 - 9x_0 = 0$

$x_0 = \frac{4,601}{9} = 0,5112\ m$

$M(x_0) = 4,601 \cdot 0,5112 + 17,798 - 9 \cdot 0,5112 \cdot \frac{0,5112}{2} =$

$= 18,974\ kN \cdot m$

Numer elementu 1 — Element utwierdzenie-utwierdzenie

EI	=	16270	
EI0	=	16270	
EI/EIO	=	1	
L	=	6	

K

12*EI/L^3	6*EI/L^2	.-12*EI/L^3	6*EI/l^2
6*EI/L^2	4*EI/L	.-6*EI/L^2	2*EI/L
.-12*EI/L^3	.-6*EI/L^2	12*EI/L^3	.-6*EI/L^2
6*EI/l^2	2*EI/L	.-6*EI/L^2	4*EI/L

K

0,05556	0,16667	-0,05556	0,16667
0,16667	0,66667	-0,16667	0,33333
-0,05556	-0,16667	0,05556	-0,16667
0,16667	0,33333	-0,16667	0,66667

	Q	P	M
	9	0	0
.T01	.q*L/2	.P*L/2	.-3*M/(2*L)
.M01	.-q*L^2/12	.-P*L/8	.M/4
.T10	.-q*L/2	.-P*L/2	.-3*M/(2*L)
.M10	.q*L^2/12	.P*L/8	.M/4
.T01	27	0	0
.M01	-27	0	0
.T10	-27	0	0
.M10	27	0	0

Re

.-T01	=	-27
.M01	=	-27
.T10	=	-27
.M10	=	27

Numer elementu 2 — Element utwierdzenie-przegub

EI	=	8090	
EI0	=	16270	
EI/EIO	=	0,4972	
L	=	5,5	

K

3*EI/L^3	3*EI/L^2	.-3*EI/L^3	
3*EI/L^2	3*EI/L	.-3*EI/L^2	
.-3*EI/L^3	.-3*EI/L^2	3*EI/L^3	

K

0,00897	0,04931	-0,00897	
0,04931	0,27122	-0,04931	
-0,00897	-0,04931	0,00897	

	Q	P	M
	0	44	0
.T01	.5*q*L/8	.11*P/16	.-9*M/(8*L)
.M01	.-q*L^2/8	.-3*P*L/16	.M/8
.T10	.-3*q*L/8	.-5*P/16	.-9*M/(8*L)
.T01	0	30,25	0
.M01	0	-45,375	0
.T10	0	-13,75	0

Re

.-T12	=	-30,25
.M12	=	-45,375
.T21	=	-13,75

Numer elementu 3 — Element przegub-utwierdzenie

EI	=	8090	
EI0	=	16270	
EI/EIO	=	0,4972	
L	=	5,5	

K

3*EI/L^3	.-3*EI/L^3	3*EI/L^2	
.-3*EI/L^3	3*EI/L^3	.-3*EI/L^2	
3*EI/L^2	.-3*EI/L^2	3*EI/L	

K

0,00897	-0,00897	0,04931	
-0,00897	0,00897	-0,04931	
0,04931	-0,04931	0,27122	

	Q	P	M
	8	0	0
.T01	.3*q*L/8	.5*P/16	.-9*M/(8*L)
.T10	.-5*q*L/8	.-11*P/16	.-9*M/(8*L)
.M10	.q*L^2/8	.3*P*L/16	.M/8
		0	
.T01	16,5	0	0
.T10	-27,5	0	0
.M10	30,25	0	0

Re

.-T23	=	-16,5
.T32	=	-27,5
.M32	=	30,25

Tablica powiązań

Numer elementu	Numer przemieszczeń			
	1	2	3	4
1	1	2	3	4
2	3	4	5	-
3	5	6	7	-

Przemieszczenia zablokowane: 4, 5

Agregacja globalnego wektora reakcji węzłowych

1			R011			-27			-27	
2			R012			-27			-27	
3			R013	R021		-27	-30,25		-57,25	
4	Ro	=	R014	R022	=	27	-45,375	=	-18,375	
5				R023	R031		-13,75	-16,5	-30,25	
6					R032			-27,5	-27,5	
7					R033			30,25	30,25	

Ro red = -18,375 / -30,25

Agregacja globalnej macierzy sztywności

				1	2	3	4	5	6	7
1				0,05556	0,16667	-0,05556	0,16667	0,00000	0,00000	0,00000
2				0,16667	0,66667	-0,16667	0,33333	0,00000	0,00000	0,00000
3				-0,05556	-0,16667	0,06452	-0,11735	-0,00897	0,00000	0,00000
4	K	=	EI	0,16667	0,33333	-0,11735	0,93789	-0,04931	0,00000	0,00000
5				0,00000	0,00000	-0,00897	-0,04931	0,01793	-0,00897	0,04931
6				0,00000	0,00000	0,00000	0,00000	-0,00897	0,00897	-0,04931
7				0,00000	0,00000	0,00000	0,00000	0,04931	-0,04931	0,27122

K red = 0,93789 / -0,04931 / 0,01793

Agregacja globalnego wektora sił węzłowych

1			0	
2			0	
3			0	
4	Pw	=	-35	Pw red = -35
5			15	15
6			0	
7			0	

P red = Pw red - Ro red

P red	=	-35	-	-18,375	=	-16,625
		15	-	-30,25	=	45,25

K red * q red = P red q red = k red^-1 * P red

0,93789	q4	-0,04931	q4	=	-16,625		0,93789	q4	+	-0,04931	q4	+	16,625	=	0
-0,04931	q5	0,01793	q5	=	45,25		-0,04931	q5	+	0,01793	q5	+	-45,25	=	0

W = 0,0143863 — W1 = 1,9332734 — W2 = 41,61949

q4 = 134,38331 / EI — q5 = 2893,0025 / EI

Element 1 R1 = K1 * q01 + R01

q01 = q1 = 0; q2 = 0; q3 = 0; q4 = 134,38331

EI	0,05556	0,16667	-0,05556	0,16667	*1/EI	0	+	-27	=	22,397218	+	-27	=	-4,602782	=	.-T01
	0,16667	0,66667	-0,16667	0,33333		0		-27		44,794436		-27		17,794436		.M01
	-0,05556	-0,16667	0,05556	-0,16667		0		-27		-22,39722		-27		-49,39722		.T10
	0,16667	0,33333	-0,16667	0,66667		134,38331		27		89,588871		27		116,58887		.M10

Element 2 R2 = K2 * q02 + R02

q02 = q3 = 0; q4 = 134,38331; q5 = 2893,0025; -

EI	0,00897	0,04931	-0,00897	0,00000	*1/EI	0	+	-30,25	=	-19,31161	+	-30,25	=	-49,56161	=	.-T12
	0,04931	0,27122	-0,04931	0,00000		134,38331		-45,375		-106,2139		-45,375		-151,5889		.M12
	-0,00897	-0,04931	0,00897	0,00000		2893,0025		-13,75		19,311613		-13,75		5,5616129		.T21
	0,00000	0,00000	0,00000	0,00000		0		0		0		0		0		0

Element 3 R3 = K3 * q03 + R03

q03 = q5 = 2893,0025; q6 = 0; q7 = 0; - = 0

EI	0,00897	-0,00897	0,04931	0,00000	*1/EI	2893,0025	+	-16,5	=	25,938387	+	-16,5	=	9,4383871	=	.-T23
	-0,00897	0,00897	-0,04931	0,00000		0		-27,5		-25,93839		-27,5		-53,43839		.T32
	0,04931	-0,04931	0,27122	0,00000		0		30,25		142,66113		30,25		172,91113		.M32
	0,00000	0,00000	0,00000	0,00000		0		0		0		0		0		0